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ПОВЫШЕНИЕ СТЕПЕНИ УСТОЙЧИВОСТИ СИСТЕМЫ ВЗАИМОСВЯЗАННЫХ ХИМИЧЕСКИХ РЕАКТОРОВ

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Предложена методика синтеза управления концентрацией и температурой реагента в химическом реакторе проточного типа. В качестве объекта управления рассматриваются взаимосвязанные химические реакторы, описываемые системой обыкновенных дифференциальных уравнений. В системе осуществляется экзотермическая реакция первого порядка. Цель управления состоит в стабилизации заданного режима функционирования последовательно соединенных химических ректоров, в повышении интенсивности протекания химических реакций и, как следствие, уменьшении времени переходных процессов за счет увеличения степени устойчивости нелинейной системы. Степень устойчивости взаимосвязанной системы химических реакторов определяется величиной старшего отрицательного характеристического показателя Ляпунова. Задача стабилизации, в контексте данной статьи, понимается как повышение степени устойчивости системы взаимосвязанных химических реакторов, при регулярных режимах в виде особых точек и периодических траекторий. Поставленная цель достигается введением в систему обратной связи по переменным состояния, которая позволяет сформировать требуемый спектр характеристических показателей Ляпунова. Коэффициенты обратной связи определяются методом модального управления на основе решения матричного алгебраического уравнения Сильвестра. С использованием методов математического моделирования проведено исследование поведения системы, которая состоит из трёх химических реакторов, соединенных последовательно. Исследование проводилось для нелинейной системы при отсутствии управляющих воздействий и при их наличии. Для определения показателя устойчивости системы взаимодействующих химических реакторов вычислены характеристические показатели Ляпунова. Результаты проведенных вычислительных экспериментов подтверждают работоспособность предложенного метода управления и достижение поставленной цели - формирование требуемой степени устойчивости процессов в химических реакторах.

Ключевые слова: химический реактор, характеристические показатели Ляпунова, модальное управление, нелинейная система

INCREASING THE DEGREE OF STABILITY OF THE SYSTEM OF INTERCONNECTED CHEMICAL REACTORS

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A method for the synthesis of controlling the concentration and temperature of a reagent in a flow-type chemical reactor is proposed. A system of interconnected chemical reactors is considered as a control object. A first-order exothermic reaction takes place in the system. The purpose

of the control is to stabilize the given mode of operation of series-connected chemical reactors, to increase the intensity of chemical reactions and, as a result, to reduce the time of transient processes by increasing the degree of stability of the nonlinear system. The degree of stability of an interconnected system of chemical reactors is determined by the value of the leading negative Lyapunov characteristic exponent. Within the context of this article, the task of stabilization is understood as an increase in the degree of stability of a system of interconnected chemical reactors under regular regimes in the form of singular points and periodic trajectories. This goal is achieved by introducing a state variables feedback into the system, which makes it possible to form the required spectrum of Lyapunov characteristic exponents. The feedback coefficients are determined by the modal control method based on the solution of the Sylvester matrix algebraic equation. Using the methods of mathematical modeling, a study was made of the behavior of a system that consists of three chemical reactors connected in series. The study was carried out for a nonlinear system in the absence of control actions and in their presence. To determine the stability index of a system of interacting chemical reactors, the Lyapunov characteristic exponents are calculated. The simulation results confirm the operability of the proposed control method and the achievement of the set goal – the formation of the required degree of process stability in chemical reactors.

Key words: chemical reactor, Lyapunov characteristic exponents, modal control, nonlinear system

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INTRODUCTION

A system of chemical reactors interacting in terms of a reagent and a refrigerant is characterized by significant nonlinearities in the main functional dependencies and the presence of uncertainty in setting the parameters of the reactors and the reactant [1]. In this case, the behavior of a nonlinear chemical system can have a complex dynamic character. The essence of such modes is the presence of non-periodic changes in the concentration and temperature of reagents, which arise from parameter variations or the effect of noise [2-5].

To control chemical systems under the action of parametric and external disturbances, a fairly large range of methods of the theory of automatic control is currently used. These include: PID controllers [6-9]; regulators providing robust stability [10-13]; adaptive controllers [14-17]. A synergistic approach based on the analytical design of aggregated controllers has become widespread [18-21].

In this paper, we consider the problem of increasing the degree of stability of a nonlinear system of interconnected flow-type chemical reactors based on the formation of the required spectrum of Lyapunov characteristic indicators by the modal control method using the solution of the Sylvester matrix algebraic equation generalized to nonlinear systems [22].

MATHEMATICAL MODEL OF A CHEMICAL REACTOR. FORMULATION OF THE STABILIZATION PROBLEM

A conceptual (meaningful) model of the functioning of a chemical reactor of an ideal mixing of a flow type with a jacket in which an exoteric reaction takes place is taken as the initial one [11]. The model of processes in a chemical reactor, built under the following assumptions, is taken as the initial one: all reagents form a single-phase system; the incoming elements of the reacting mixture are instantly mixed with the contents of the reactor and the state of the mixture (concentration, temperature of the reagents) at each moment of time will have the same values throughout the entire volume of the reactor; heat removal from the reactors is carried out through the jacket and all reactors are non-isothermal (heat removal through the reactor wall is not instantaneous); the reactors are of flow type. An exothermic reaction of the first order is carried out in the system, the reaction rate increases with increasing temperature and obeys the Arrhenius law.

The mathematical model of a chemical reactor is described by a system of three nonlinear differential equations and is of a fairly general nature applicable to the study of the behavior of some arbitrary chemical reactions that satisfy the assumptions listed above,

$$\dot{C}_i = \frac{F_i}{V_i}(C_{i-1} - C_i) - K\exp\left(\frac{-E_i}{R}T_i^{-1}\right)C_i \qquad (1a)$$

$$\dot{T}_i = \frac{F_i}{V_i}(T_{i-1} - T_i) - \frac{L_i S_i}{\rho_i \gamma_i V_i}(T_i - T_{Ci}) + + \frac{K\Delta H_i}{\rho_i \gamma_i}\exp\left(\frac{-E_i}{R}T_i^{-1}\right)C_i, \quad (1b)$$

$$\dot{T}_{Ci} = \frac{F_C}{V_C}(T_{C(i+1)} - T_{Ci}) + \frac{L_i S_i}{\rho_C \gamma_C V_C}(T_i - T_{Ci}). \quad (1c)$$

In the system of equations (1) the following designations are used: F_i – space velocity of the substance supply to the i-th reactor; C_{i-1}, C_i – initial and final concentration of the substance in the i-th reactor; T_{i-1}, T_i – temperature at the inlet and outlet of the i-th reactor; F_C – space velocity of the coolant supply to the reactors; T_{Ci} is the temperature of the coolant in the i-th reactor. Equation (1a) follows from the law of conservation of mass and determines the rate of change in the reagent concentration in the i-th reactor. Equation (1b) follows from the conservation of energy and determines the rate of temperature change in the i-th reactor. Equation (1c) determines the rate of change in the temperature of the refrigerant in the i-th reactor.

Here V_i is the working volume of the i-th reactor; K – multiplier in the Arrhenius law; E_i is the activation energy of the reaction in the i-th reactor; R – universal gas constant; $\dot{C}_i = dC_i/dt$ – time derivative; ΔH_i – thermal effect of the reaction; ρ_i – specific mass heat capacity of the reagent; γ_i – reagent density; L_i – heat transfer coefficient; S_i – heat exchange surface area; ρ_C – specific mass heat capacity of the refrigerant; γ_C – the density of the refrigerant.

If we introduce the vector of phase coordinates:

$$z(t) = (z_1(t) = C_1,\ z_2(t) = T_1,\ z_3(t) = T_{C_1},\ z_4(t) = C_2,$$
$$z_5(t) = T_2,\ z_6(t) = T_{C_2},\ z_7(t) = C_3,\ z_8(t) = T_3,\ z_9(t) = T_{C_3})^T,$$

then the system of nonlinear differential equations describing three series-connected chemical reactors has the form:

$$\dot{z}_1(t) = \frac{F}{V_1}(C_0 - z_1(t)) - K\exp\left(\frac{-E_1}{R}z_2^{-1}(t)\right)z_1(t)$$
$$\dot{z}_2(t) = \frac{F}{V_1}(T_0 - z_2(t)) - \frac{L_1 S_1}{\rho_1\gamma_1 V_1}(z_2(t) - z_3(t)) + + \frac{K\Delta H_1}{\rho_1\gamma_1}\exp\left(\frac{-E_1}{R}z_2^{-1}(t)\right)z_1(t),$$
$$\dot{z}_3(t) = \frac{F_C}{V_C}(z_6(t) - z_3(t)) + \frac{L_1 S_1}{\rho_C\gamma_C V_C}(z_2(t) - z_3(t))$$
$$\dot{z}_4(t) = \frac{F}{V_2}(z_1(t) - z_4(t)) - K\exp\left(\frac{-E_2}{R}z_5^{-1}(t)\right)z_4(t)$$
$$\dot{z}_5(t) = \frac{F}{V_2}(z_2(t) - z_5(t)) - \frac{L_2 S_2}{\rho_2\gamma_2 V_2}(z_5(t) - z_6(t)) + + \frac{K\Delta H_2}{\rho_2\gamma_2}\exp\left(\frac{-E_2}{R}z_5^{-1}(t)\right)z_4(t),$$
$$\dot{z}_6(t) = \frac{F_C}{V_C}(z_9(t) - z_6(t)) + \frac{L_2 S_2}{\rho_C\gamma_C V_C}(z_5(t) - z_6(t)) \quad (2)$$
$$\dot{z}_7(t) = \frac{F}{V_3}(z_4(t) - z_7(t)) - K\exp\left(\frac{-E_3}{R}z_8^{-1}(t)\right)z_7(t)$$
$$\dot{z}_8(t) = \frac{F}{V_3}(z_5(t) - z_8(t)) - \frac{L_3 S_3}{\rho_3\gamma_3 V_3}(z_8(t) - z_9(t)) + + \frac{K\Delta H_3}{\rho_3\gamma_3}\exp\left(\frac{-E_3}{R}z_8^{-1}(t)\right)z_7(t),$$
$$\dot{z}_9(t) = \frac{F_C}{V_C}(T_{C_0} - z_9(t)) + \frac{L_3 S_3}{\rho_C\gamma_C V_C}(z_8(t) - z_9(t))$$

The scalar system of differential equations (2) can be written in the vector-matrix form:

$$\dot{z}(t) = F(z(t)), \quad z(t_0) = z_0 \quad (3)$$

where $F(z(t)) = (f_i(z(t)))_{i=1}^{9}:\ R^9 \to R^9$ is a vector function describing the dynamic properties of the investigated system of chemical reactors.

Statement of the problem of stabilization (increasing the degree of stability) of an interconnected system of reactors. Let the model of an interconnected system of reactors be described by an autonomous vector differential equation (3).

One of the features of nonlinear systems is the possibility of the emergence of regimes characterized by instability of trajectories. The quantitative measure of this instability is the characteristic exponents, originally introduced by Lyapunov. Formally, the characteristic Lyapunov exponent is introduced as follows. The characteristic indicator of a function $z(t)$ is a number (or symbol $\pm\infty$) defined as:

$$\lambda(z) \equiv \overline{\lim_{t\to\infty}}\left(t^{-1}\ln\|z(t)\|\right).$$

The characteristic Lyapunov exponent of the function $z(t)$ is the result of comparing the rate of change of the function $z(t)$ at $t\to\infty$ with the exponent $\exp\{\alpha t\}$. Among the entire set of Lyapunov characteristic exponents, the largest (senior) exponent $\lambda_1 = \lambda_{max}$ is the most important. The set of characteristic exponents, sorted in descending order $\lambda_1 \ge \lambda_2 \ge \ldots \ge \lambda_n$, is called the Lyapunov spectrum of a nonlinear dynamical system.

For the trajectories $z(t, z_0)$ of system (3), one of three possibilities can be fulfilled:

– either the trajectory $z(t,z_0) = z^0$ is a rest point or a state of equilibrium, for it the signature of the Lyapunov spectrum (signs of the characteristic exponents) has the form:

$$\underbrace{(-,-,-,\ldots,-,-,-)}_{n} \quad (4a)$$

corresponding to a stable focus or node;

– or the trajectory $z(t,z_0)$ is a periodic solution, in which case there exists a number $T > 0$ such that $z(t+T, z_0) \equiv z(t, z_0)$, for which the signature of the Lyapunov spectrum has the form:

$$(0, \underbrace{-,-,...,-,-,-}_{n-1}) \quad (4b)$$

corresponding to a stable limit cycle;

– or for any $t_1 \neq t_2$ $z(t_1, z_0) \neq z(t_2, z_0)$, in which case the trajectory is not closed, and the signature of the Lyapunov spectrum can be of the form:

$$(\underbrace{+,...,+}_{s}, -, ..., -) \quad (4c)$$

The task of controlling an interconnected system of reactors is to stabilize a singular point or limit cycle and increase the degree of stability, which is determined by the value of the leading Lyapunov characteristic exponent [22]. Control synthesis is reduced to finding feedback on the state of a nonlinear system:

$$u(t) = u(z(t)), \quad u(z(t)) = -Kz(t). \quad (5)$$

In this case, the Lyapunov spectrum of a closed nonlinear system must coincide with the spectrum of the form (4a) or (4b).

Thus, the task of stabilizing a nonlinear system of interconnected reactors consists in shifting the spectrum of Lyapunov characteristic exponents by introducing feedback on the phase coordinates of the system and creating stable rest points or a limit cycle by forming negative Lyapunov characteristic exponents in a closed system.

CONTROL OF THE LAPUNOV CHARACTERISTIC EXPONENTS OF A SYSTEM OF INTERACTING REACTORS

Synthesis of control of a nonlinear system by introducing feedback consists in changing the spectrum of Lyapunov characteristic exponents to achieve the desired result – increasing the degree of stability of the nonlinear system.

To solve the problem of changing the spectrum of Lyapunov characteristic exponents, the fact that they are determined by the eigenvalues of the Jacobian matrix of the linearized system is used. A change in the eigenvalues of the Jacobian matrix, the real parts of which determine the characteristic exponents of the linearized system, entails a change in the Lyapunov characteristic exponents of the nonlinear system. The desired eigenvalues can be assigned to the Jacobian matrix using the modal control synthesis technique based on solving the matrix algebraic Sylvester equation.

The validity of this approach is substantiated by the theorem on the topological equivalence of a nonlinear system and a linearized model [23, 24]. It follows from the theorems that equilibrium states, periodic and non-closed trajectories of a linearized system are mapped, respectively, into equilibrium states, periodic and non-closed trajectories of a nonlinear system. The theorem is valid for hyperbolic systems that do not have purely imaginary eigenvalues.

Linearization of an interacting system of reactors. If the following conditions are met:

$F(z(t)) = (f_i(z(t)))_{i=1}^{9}$ is a vector function that satisfies the conditions for the existence of a solution to equation (3);

$f_i(z(t))$ are real continuous functions, then there is a partial derivative of the function with respect $F(z(t))$ to the vector argument $z(t)$:

$$J(z) = \partial F(z(t)) / \partial z(t)\big|_{z^*}, \; J(z) = (j_{ik})_{i,k=1}^{n,n}, \; j_{ik} = \partial f_i(z(t)) / \partial z_k\big|_{z^*} \quad (6)$$

and system (3) corresponds to the linear differential equation:

$$\dot{y}(t) = J(z^*)y(t) + By(t) \quad (7)$$

Here, $J(z^*)$ is the Jacobian matrix of the vector function $F(z(t))$.

Synthesis of feedback for a nonlinear system. When solving the stabilization problem, the eigenvalues of the Jacobian matrix of system (7) with control (5) must be negative:

$$\bar{v}(\tilde{J}(z^*)) = \alpha \cdot \mathrm{Re}(v(J(z^*)))$$

where $v(J(z^*))$ are the eigenvalues of the Jacobian matrix of the original system; α is a coefficient that affects the shift of the eigenvalues of the matrix along the real axis of the complex plane. When the system stabilizes, the coefficient α is chosen negative.

Based on the required eigenvalues of the Jacobian matrix of the closed system (7), the feedback coefficients are calculated. The spectrum of Lyapunov characteristic exponents of a closed system of the system is checked for compliance with signature (4a).

Synthesis of control of a linearized system. The problem of positioning the poles of the system is considered, in which the determination of the controller parameters is reduced to solving the matrix Sylvester equation [25].

For a linearized dynamic system (7), it is necessary to find a stabilizing controller in the form (5) such that the spectrum of the closed system:

$$\dot{y}(t) = J(z^*)y(t) - BKy(t)$$

coincided with the prescribed spectrum, which is given by the set $\mu = \{\mu_1, ..., \mu_n\}$,

$$\rho(\tilde{J}(z)) = \rho(-\Phi).$$

Here, $\Phi = \mathrm{diag}\,(\mu_i)_{i=1}^{n} \in \mathrm{R}^{n\times n}$ is a matrix with numbers μ_i on its main diagonal.

The task of finding the matrix K, which determines the "depth" of the feedback on the state vector, is calculated by the formula:

$$K = GP^{-1} \tag{8}$$

where P is the solution of the Sylvester matrix equation

$$J(z^*)P + P\Phi = BG \tag{9}$$

For the dynamical system (7), the conditions for the existence of a solution to the pole placement problem are contained in the following theorem.

Theorem. Let system (7) satisfy the following conditions:

1) matrix $G \in \mathrm{R}^{m\times n}$ of full rank $\Leftrightarrow \mathrm{rank}G = q$, $q = \min\{m, n\}$;

2) the matrix pair (J, B) is controllable $\Leftrightarrow \mathrm{rank}D = n$, where the controllability matrix is $D = \left(B \middle| JB \middle| ... \middle| J^{n-1}B\right) \in \mathrm{R}^{n\times mn}$;

3) the matrix pair (G, Φ) is observable $\Leftrightarrow \mathrm{rank}\Omega = n$, where the observability matrix is $\Omega = \left(G^T \middle| \Phi^T G^T \middle| ... \middle| (\Phi^T)^{n-1} G^T\right) \in \mathrm{R}^{n\times mn}$;

4) the spectra of matrices J and Φ do not intersect $\rho(J) \cap \rho(\Phi) = \varnothing$;

5) the numbers μ_i $(i = \overline{1,n})$ defining the prescribed spectrum $\mu = \{\mu_1, ..., \mu_n\}$ are pairwise different $\mu_i \cap \mu_j = \varnothing$,

then there is a control (5) such that the closed-loop matrix has a spectrum that coincides with the spectrum of the reference matrix $(-\Phi)$.

The controller parameters are determined from relation (8), where the matrix P is the solution of the Sylvester equation (9).

By substituting into the Sylvester equation (9) the matrix G associated by equation (8) with the desired matrix K, we obtain:

$$J(z^*)P + P\Phi = BKP$$

or after obvious transformations we have:

$$P^{-1}(J(z^*) - BK)P = -\Phi.$$

The last equality means that the matrix $\tilde{J} = \left(J(z^*) - BK\right)$ is similar to the matrix $(-\Phi)$. Similar matrices have the same eigenvalues (the same spectra), so $\rho(\tilde{J}) = \rho(-\Phi)$.

It follows from the theorem that to determine the stabilizing controller that provides negative eigenvalues in system (7), it is necessary to solve the Sylvester matrix equation (9) and find the controller parameters using relation (8). Regulator (8), by virtue of the topological equivalence theorem [23] of a linearized system (7) and a nonlinear system (3), is also stabilizing for a nonlinear system of interacting chemical reactors.

INVESTIGATION OF A SYSTEM OF INTERCONNECTED REACTORS

We will illustrate the proposed method of control synthesis and analysis of the properties of a closed system for a system consisting of three series-connected chemical reactors.

Properties of the system without control. The coordinates of the singular (stationary) point of system (3) obtained as a result of solving the system of nonlinear equations $F(z(t), u(t)) = 0$ are equal to:

$$O_1 = (z_1 = 5.98, z_2 = 301.37, z_3 = 301.37, z_4 = 5.96,$$
$$z_5 = 300.77, z_6 = 300.77, z_7 = 5.95, z_5 = 300.19, z_6 = 300.18)$$

The Jacobian matrix of system (3) at a singular point O_1 is equal to:

$$J(z^*, u^*) = \begin{bmatrix} -0.03 & -3.13\cdot10^{-5} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -3.27\cdot10^{-3} & -0.07 & 0.04 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 35.97 & -35.99 & 0 & 0 & 0.02 & 0 & 0 & 0 \\ 0.03 & 0 & 0 & -0.03 & -3.03\cdot10^{-5} & 0 & 0 & 0 & 0 \\ 0 & 0.03 & 0 & -3.16\cdot10^{-3} & -0.07 & 0.04 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 35.97 & -35.99 & 0 & 0 & 0.02 \\ 0 & 0 & 0 & 0.03 & 0 & 0 & -0.03 & -2.94\cdot10^{-5} & 0 \\ 0 & 0 & 0 & 0 & 0.03 & 0 & -3.05\cdot10^{-3} & -0.07 & 0.04 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 35.97 & -35.99 \end{bmatrix} \tag{10}$$

The elements of the Jacobian matrix (10) are obtained for the following values of the parameters $V_i = 0.08\ cm^3$, $K = 2000\ s^{-1}$, $E_i = 42290\ J/mol$, $\rho_i = 4190000\ J/kgK$, $R = 8.314\ J/mol$, $\Delta H_i = -146538\ J/mol$, $S_i = 0.09\ m^2$, $\gamma_i = 0.001\ kg/m^3$, $L_i = 167.472\ J/m^2 s$, $\rho_C = 4190\ J/kgK$, $\gamma_C = 0.001\ kg/cm^3$, $F = 0.0025\ м^3/c$, $F_c = 0.002\ м^3/c$ and initial conditions $C_i(0) = 6\ моль/м^3$, $T_i(0) = 302\ \mathrm{K}$, $T_{ci}(0) = 298\ \mathrm{K}$.

The spectrum of Lyapunov characteristic exponents of the system (3) is equal to

$$\rho(F) = \{\lambda_1 = -0.01; \lambda_2 = -0.01; \lambda_3 = -0.03; \lambda_4 = -0.13; \lambda_5 = -0.20;$$
$$\lambda_6 = -0.4; \lambda_7 = -30.80; \lambda_8 = -31.10; \lambda_9 = -31.23\},$$

which indicates the weak stability of the system.

Properties of a system with control. Let the matrix B of system (3) equal to:

$$B = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \tag{11}$$

describes the effect of the control actions on the dynamics of processes in a system of series-connected reactors.

The controllability of the system (7) is determined by the controllability of the matrix pair. In accordance with the formula of condition 2 of the theorem, the controllability matrix $D \in \mathrm{R}^{9\times(9\times9)}$ is calculated. Its full rank is nine, which indicates the controllability of the chemical reactor system. The matrix J is defined by expression (10), and the matrix B is defined by expression (11).

To check the observability of the system, in accordance with the formula of condition 3 of the theorem, the observability matrix $\Omega \in \mathrm{R}^{9\times(9\times9)}$ is calculated. Its full rank is nine, which indicates the observability of the system. Here the matrices G and Φ are equal

$$G = \begin{bmatrix} 1&0&0&0&0&0&0&0&0 \\ 0&1&0&0&0&0&0&0&0 \\ 0&0&1&0&0&0&0&0&0 \\ 0&0&0&1&0&0&0&0&0 \\ 0&0&0&0&1&0&0&0&0 \\ 0&0&0&0&0&1&0&0&0 \\ 0&0&0&0&0&0&1&0&0 \\ 0&0&0&0&0&0&0&1&0 \\ 0&0&0&0&0&0&0&0&1 \end{bmatrix}, \quad \Phi = \begin{bmatrix} 2&0&0&0&0&0&0&0&0 \\ 0&2{,}5&0&0&0&0&0&0&0 \\ 0&0&3&0&0&0&0&0&0 \\ 0&0&0&4&0&0&0&0&0 \\ 0&0&0&0&4{,}5&0&0&0&0 \\ 0&0&0&0&0&5&0&0&0 \\ 0&0&0&0&0&0&31&0&0 \\ 0&0&0&0&0&0&0&32&0 \\ 0&0&0&0&0&0&0&0&33 \end{bmatrix}.$$

The fulfillment of the controllability and observability conditions make it possible to find a solution to the Sylvester matrix equation (9) and calculate the feedback coefficient using the formula (8), which stabilizes the system of chemical reactors at the operating point,

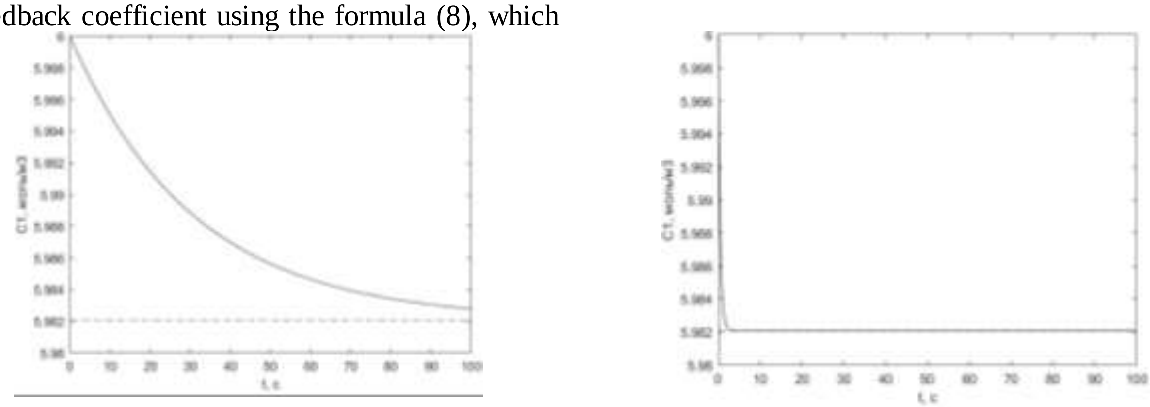
$$K = \begin{bmatrix} -2.03 & -2.52\cdot10^{-5} & 6.14\cdot10^{-8} & 2.24\cdot10^{-15} & -4.09\cdot10^{-13} & 2.45\cdot10^{-11} & -7.39\cdot10^{-17} & 5.77\cdot10^{-16} & -8.77\cdot10^{-15} \\ -4.08\cdot10^{-3} & -2.57 & 0.04 & -1.05\cdot10^{-9} & 1.02\cdot10^{-6} & -7.28\cdot10^{-6} & 2.73\cdot10^{-15} & -2.36\cdot10^{-11} & -7.54\cdot10^{-10} \\ -7.71\cdot10^{-4} & 36.44 & -38.99 & -1.73\cdot10^{-7} & 2.11\cdot10^{-4} & 0.02 & -9.60\cdot10^{-13} & 8.41\cdot10^{-9} & -2.24\cdot10^{-6} \\ 0.06 & -1.13\cdot10^{-7} & 2.74\cdot10^{-9} & -4.03 & -2.70\cdot10^{-5} & 2.39\cdot10^{-8} & -1.41\cdot10^{-15} & 1.45\cdot10^{-13} & 2.25\cdot10^{-12} \\ -7.58\cdot10^{-5} & 0.06 & -1.36\cdot10^{-4} & -3.55\cdot10^{-3} & -4.57 & 0.04 & 4.75\cdot10^{-12} & -4.74\cdot10^{-8} & -9.95\cdot10^{-6} \\ -7.35\cdot10^{-5} & 0.02 & -1.32\cdot10^{-4} & -3.57\cdot10^{-4} & 36.42 & -40.99 & -9.65\cdot10^{-9} & 9.25\cdot10^{-5} & 0.01 \\ 3.64\cdot10^{-3} & -3.62\cdot10^{-8} & 5.66\cdot10^{-10} & 0.24 & -1.01\cdot10^{-6} & 3.15\cdot10^{-9} & -31.03 & -2.85\cdot10^{-5} & 1.17\cdot10^{-9} \\ -1.75\cdot10^{-5} & 2.16\cdot10^{-3} & -1.52\cdot10^{-5} & -1.85\cdot10^{-4} & 0.22 & -1.96\cdot10^{-4} & -3.16\cdot10^{-3} & -32.08 & 0.04 \\ -1.74\cdot10^{-5} & 2.16\cdot10^{-3} & -1.52\cdot10^{-5} & -1.71\cdot10^{-4} & 0.17 & -1.93\cdot10^{-4} & -5.37\cdot10^{-5} & 36.50 & -68.99 \end{bmatrix}.$$

The spectrum of Lyapunov characteristic exponents of the system with synthesized control is equal to:

$$\bar{\rho}(F(z,u) = \{\bar{\lambda}_1 = -2.06; \bar{\lambda}_2 = -2.56; \bar{\lambda}_3 = -4.30;$$
$$\bar{\lambda}_4 = -4.46; \bar{\lambda}_5 = -31.22; \bar{\lambda}_6 = -32.23; \bar{\lambda}_7 = -74.73;$$
$$\bar{\lambda}_8 = -76.95; \bar{\lambda}_9 = -104.97\}.$$

and the value of the senior Lyapunov characteristic exponent indicates an increase in the degree of stability.

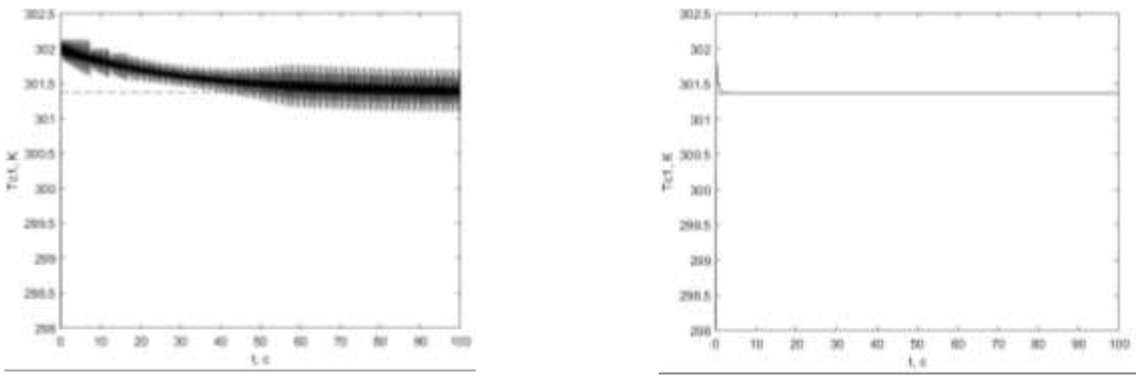
Fig. 1 and 2 show time diagrams of transients in the components of the phase vector of the first reactor: the concentration of the substance $z_1(t) = C_1$ and the temperature of the refrigerant $z_3(t) = T_{C_1}$ in the system of chemical reactors without control and with control. The horizontal dotted line corresponds to the operating point of the chemical reactor.



a b

Fig. 1. Transient processes in the system by components $z_1(t) = C_1$; a) without control, b) with control

Рис. 1. Переходные процессы в системе по компонентам $z_1(t) = C_1$; a) без управления, b) с управлением



a b

Fig. 2. Transient processes in the system by components $z_3(t) = T_{C_1}$; a) without control, b) with control

Рис. 2. Переходные процессы в системе по компонентам $z_3(t) = T_{C_1}$; a) без управления, b) с управлением

From the above time diagrams, it follows that the transition time in a controlled system is reduced approximately tenfold, which is associated with a decrease in the senior Lyapunov characteristic index and an increase in the degree of stability of the nonlinear system.

CONCLUSIONS

A technique for increasing the degree of stability of a nonlinear system of series-connected chemical reactors is proposed, based on the use of the modal control method based on the solution of the linear Sylvester matrix equation. The application of the proposed technique is considered on the example of three series-connected chemical reactors. The computational experiment confirms the shift of the spectrum of Lyapunov characteristic exponents in the direction of increasing the degree of stability of the nonlinear system closed by the synthesized control.

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The authors declare the absence a conflict of interest warranting disclosure in this article.

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