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МОДЕЛИРОВАНИЕ ПРОЦЕССОВ ТЕПЛОПРОВОДНОСТИ И ДИФФУЗИИ В ТЕЛАХ КАНОНИЧЕСКОЙ ФОРМЫ С ПРИМЕНЕНИЕМ МЕТОДА «МИКРОПРОЦЕССОВ» ДЛЯ ОБЛАСТИ МАЛЫХ ЗНАЧЕНИЙ ЧИСЛА ФУРЬЕ

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В работе представлены краевые задачи теплопереноса (массопереноса) для пластины, шара и цилиндра при равномерном начальном распределении температур в безразмерных переменных. Целью настоящей работы является изложение принципов получения решений краевых задач для более общих граничных условий – условий третьего рода. В ходе преобразования Лапласа, по временной переменной Фурье, в работе показаны зависимости в области изображений для тел канонической формы: пластины, цилиндра и шара. Также приведены решения в области изображений и комплексных переменных. Указано, что для пластины граничные условия являются общими для задач теплопроводности и диффузии: при критерии Био, стремящемся к нулю, они переходят в условия второго рода, а при критерии Био, стремящемся к бесконечности, переходят в условия первого рода. Для решения краевых задач тепло- и массопереноса для области малых значений чисел Фурье в работе указаны свойства гиперболических функций, в результате чего приведены окончательные решения линейных неоднородных дифференциальных уравнений второго порядка. В работе представлены номограммы безразмерной температуры поверхности тела в зависимости от значений чисел Био и Фурье при конкретных значениях числа Био для поверхностей пластины, шара и цилиндра, которые позволяют с помощью простых геометрических операций исследовать функциональные зависимости без громоздких вычислений при малых значениях числа Фурье, что способствует исключению ошибки при реализации методов расчета с использованием «зонального» метода и метода «микропроцессов».

Ключевые слова: термическая обработка, тепломассоперенос, пластина, цилиндр, сфера, «зональный» метод, метод «микропроцессов», малые значения числа Фурье

MODELING OF HEAT CONDUCTION AND DIFFUSION PROCESSES IN CANONICAL BODIES USING THE "MICRO-PROCESSES" METHOD FOR THE RANGE OF LOW FOURIER NUMBERS

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This paper presents a number of boundary value problems of heat transfer (mass transfer) for a plate, a sphere and a cylinder with a uniform initial temperature distribution in dimensionless variables. The purpose of this paper is to present the principles of obtaining solutions for boundary value problems for more general boundary conditions – the third-kind boundary conditions. When applying the Laplace transform and using the Fourier transform, we show in this paper the dependences in the area of images for the bodies of canonical shape: a plate, a cylinder, and a sphere. Several solutions in the area of images and complex variables have also been given. It is pointed out that the boundary conditions for a plate are common for the problems of heat conduction and diffusion. If the Biot number tends towards zero, they change into the second-kind boundary conditions. And if the Biot number tends towards infinity, they change into the conditions of the first kind. To solve the boundary value problems of heat and mass transfer in the area of small values of Fourier numbers, we have indicated the properties of hyperbolic functions and presented the final solutions to inhomogeneous second-order differential equations. The paper presents some nomograms of dimensionless body surface temperature depending on the values of the Biot and Fourier numbers for specific values of the Biot number for the surfaces of a plate, sphere and cylinder, which allow us, by means of simple geometric operations, to study the functional dependences without cumbersome calculations for small values of the Fourier number, which contributes to eliminating errors in implementing the calculation methods using the "zonal" and "micro-processes" methods.

Key words: heat treatment, heat and mass transfer, plate, cylinder, sphere, "zonal" method, "micro-processes" method, low Fourier numbers

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INTRODUCTION

Our paper [1] considers some cases of synthesis of mathematical models of heat and mass transfer processes in canonical bodies (plate, cylinder, sphere). The issue has been considered with regard to the problems of thermal conductivity and diffusion in solids. At the same time, to simplify calculations (transformations), we consider some problems with boundary conditions of the first kind [2].

In this case, the law of change in the transfer potential (heat and mass transfer) at the boundary of a solid body with its environment is set.

In [1], expressions for calculating temperature fields depending on the thermophysical parameters, as well as the fields of mass transfer potentials, are given.

In the fundamental monograph of Academician A.V. Lykov and Professor Yu.A. Mikhailov [2], the solutions to many problems of thermal conductivity and diffusion for bodies of various configurations, including canonical bodies, have been provided. In this respect, we can note two fundamental points. First, the solutions obtained in the form of Fourier series are typical for problems with uneven initial distributions of heat and mass transfer potentials. In the case of uniform initial distributions, special cases for uniform initial conditions are easily obtained from the solutions previously obtained. However, no solutions for low Fourier numbers are given. (Fo, Fo_m<0,1).

At the same time, as it has been multiple times noted in professional publications [3], the solutions in the form of Fourier series have an "unpleasant" feature: as the process time decreases, the numerical values of the Fourier criteria that characterize the similarity of nonstationary heat and mass transfer processes (Fo_n,

Fo_m) also decrease. This, in turn, leads to an increase in the number of members of the infinite series and to calculation error accumulation. The principal values of these factors are obtained when implementing calculation methods using the "zonal" method [6-9] and the "micro-processes" method [3].

METHODS OF THEORETICAL ANALYSIS

In the dimensionless variables assigned, the boundary value problem of heat transfer will take the following form:

$$\frac{\partial\theta(\bar{x},Fo)}{\partial Fo}=\frac{\partial^2\theta(\bar{x},Fo)}{\partial\bar{x}^2}; \quad Fo\ge 0; \quad 0\le\bar{x}\le 1; \tag{1}$$

$$\theta(\bar{x},0)=1; \tag{2}$$

$$\frac{\partial\theta(0,Fo)}{\partial\bar{x}}=0; \tag{3}$$

$$\frac{\partial\theta(1,Fo)}{\partial\bar{x}}=-Bi\cdot\theta(1,Fo). \tag{4}$$

$$\theta(\bar{x},Fo)=\frac{\theta_c-\theta(x,\tau)}{\theta_c-\theta_0}; Fo=\frac{\alpha\tau}{R^2}; Bi=\frac{\alpha R}{\lambda}; \bar{x}=\frac{x}{R} \tag{5}$$

where α is the heat transfer coefficient, $\frac{W}{м^2\cdot K}$, $\theta(x,\tau)$ is the value of temperature, θ_c – ambient temperature, (K); θ_0 – initial material temperature, (K); R – half plate thickness, cylinder radius, spheres, (m); λ – thermal conductivity of body material, ($\frac{W}{m\cdot K}$). Boundary condition (3) is a symmetry condition. Note, however, that it can also be successfully used to solve problems of thermal conductivity of a plate on a thermally insulated substrate.

Accordingly, for mass transfer problems, by analogy, we can write:

$$\theta_m(\bar{x},Fo_m)=\frac{\theta_{0,m}-\theta_m(x,\tau)}{\theta_{0,m}-\theta_{c,m}}; Fo_m=\frac{k\tau}{R^2}; Bi_m=\frac{\beta R}{k}; \bar{x}=\frac{x}{R} \tag{6}$$

where β is the mass transfer coefficient, m/s; k is the mass conductivity coefficient, m^2/s, $\theta_m(x,\tau)$ is the value of mass contents, $\theta_{0,m}$ – mass content of solid phase at the initial moment of time, (kg/kg); $\theta_{c,m}$ – equilibrium mass content at the interface, (kg/kg).

Applying to the system of equations (1) – (4) Laplace transformations [10] with respect to the time variable Fo [3], we can write:

$$\frac{d^2\theta(\bar{x},s)}{d\bar{x}^2}-s\cdot\theta(\bar{x},s)+1=0; \tag{7}$$

$$\frac{d\theta(0,s)}{d\bar{x}}=0; \tag{8}$$

$$\frac{d\theta(1,s)}{d\bar{x}}=-Bi\cdot T(1,s). \tag{9}$$

The equation (7) is a linear inhomogeneous second-order differential equation [11].

The return from the area of images to the area of originals is performed by the formula in accordance with the second decomposition theorem [11]:

$$\theta(\bar{x},Fo)=L^{-1}\left[\theta(\bar{x},s)\right]=\frac{\varphi(0)}{\psi'(0)}+\sum_{n=1}^{\infty}\frac{\varphi(s_n)}{\psi'(s_n)}\exp(s_nFo). \tag{10}$$

Here: $\varphi(s)$ and $\psi'(s)$ are, respectively, the functions that appear in the numerator and denominator (11).

When solving this equation with boundary conditions (8), (9) and omitting any simple but cumbersome transformations, we write down the solution in the image domain with respect to the complex variable s:

$$\theta(\bar{x},s)=\frac{ch\left(\sqrt{s}\bar{x}\right)}{\sqrt{s}\cdot Д(s)}\cdot\left(\sqrt{s}\cdot ch\sqrt{s}+Bi\cdot sh\sqrt{s}\right)\int_0^1 ch\left(\sqrt{s}\xi\right)d\xi+ \\ -\frac{Д(s)}{\sqrt{s}\cdot Д(s)}\int_0^{\bar{x}} sh\sqrt{s}(\bar{x}-\xi)d\xi. \tag{11}$$

Here: $Д(s)=Bi\cdot ch\sqrt{s}+\sqrt{s}\cdot sh\sqrt{s}=0.$ (12)

Let us pay attention to the fact that the boundary condition (9) is common for the problems of thermal conductivity and diffusion. For $Bi\to 0$, it passes into a condition of the second kind, and for $Bi\to\infty$, it passes into a condition of the first kind [2].

As a result of mathematical transformations performed (10), it takes the following final form:

$$\theta(\bar{x},Fo)=2\sum_{n=1}^{\infty}\frac{\sin\mu_n\cdot\cos(\mu_n\bar{x})\cdot(\mu_n\cdot\cos\mu_n+Bi\cdot\sin\mu_n)}{\mu_n\left[(Bi+1)\sin\mu_n+\mu_n\cdot\cos\mu_n\right]}\exp\left(-\mu_n^2Fo\right). \tag{13}$$

In the expression obtained, the quantity μ_n defines the set of roots of the characteristic equation:

$$ctg\mu_n=\frac{\mu_n}{Bi} \tag{14}$$

Similarly, for the boundary value problem for mass transfer, the following expression will be provided:

$$\theta(\bar{x},Fo_m)=2\sum_{m=1}^{\infty}\frac{\sin\mu_m\cdot\cos(\mu_m\bar{x})\cdot(\mu_m\cdot\cos\mu_m+Bi_m\cdot\sin\mu_m)}{\mu_m\left[(Bi_m+1)\sin\mu_m+\mu_m\cdot\cos\mu_m\right]}\exp\left(-\mu_m^2Fo_m\right). \tag{15}$$

In the dimensionless variables assigned, the boundary value problem of heat transfer with uniform initial temperature distribution in a cylinder will take the following form:

$$\frac{\partial\theta(\bar{r},Fo)}{\partial Fo}=\frac{\partial^2\theta(\bar{r},Fo)}{\partial\bar{r}^2}+\frac{1}{\bar{r}}\cdot\frac{\partial\theta(\bar{r},Fo)}{\partial\bar{r}}; \; Fo>0; \; 0\le\bar{r}\le 1; \tag{16}$$

$$\theta(\bar{r},0)=1. \tag{17}$$

The boundary conditions for a problem of heat transfer with uniform initial temperature distribution in a cylinder are similar (8), (9).

$$\theta(\bar{r},Fo)=\frac{\theta_C-\theta(r,\tau)}{\theta_C-\theta_0}; \quad \bar{r}=\frac{r}{R}; \quad Fo=\frac{a\tau}{R^2}; \quad Bi=\frac{\alpha R}{\lambda}. \tag{18}$$

Accordingly, for mass transfer problems, by analogy, the subscript “m” will be added.

Again, omitting any cumbersome transformations, we present down the solution in the image domain with respect to the complex variable *s*:

$$\theta(\bar{r},s)-\frac{1}{s}=-\frac{Bi\cdot I_0\left(\sqrt{s}\bar{r}\right)}{s\left[\sqrt{s}I_0'\left(\sqrt{s}\right)+BiI_0\left(\sqrt{s}\right)\right]}. \qquad (19)$$

Here: $I_0(\sqrt{s})$, $I_0(\sqrt{s}\bar{r})$, $I_1(\sqrt{s})$, $I_1'(\sqrt{s})$ – Bessel functions of the first kind from a purely imaginary argument of zero order (a modified Bessel function of the first kind of zero order) [12-14].

Applying the inverse Laplace transform procedure [12] to the obtained expression again, we obtain a solution in the area of originals for a cylinder:

$$\theta(\bar{r},Fo)=2Bi\sum_{n=1}^{\infty}\frac{J_0(\mu_n\bar{r})}{J_0(\mu_n)\left(Bi^2+\mu_n{}^2\right)}\exp\left(-\mu_n^2Fo\right). \qquad (20)$$

Here: $J_0(\mu_n)$, $J_0(\mu_n\bar{r})$ are the usual Bessel functions defined in professional publications, for example [13, 14].

Again, we note that the solution of the mass transfer problem will be similar. In this case, the quantity *"m"* becomes the subscript for the quantities μ, Bi, Fo.

In the dimensionless variables assigned, the boundary value problem of heat transfer with uniform initial temperature distribution for a sphere will take the following form:

$$\frac{\partial\theta(\bar{r},Fo)}{\partial Fo}=\frac{\partial^2\theta(\bar{r},Fo)}{\partial\bar{r}^2}+\frac{2}{r}\cdot\frac{\partial\theta(\bar{r},Fo)}{\partial\bar{r}};\quad Fo>0;\quad 0\le\bar{r}\le 1; \qquad (21)$$

$$\theta(\bar{r},0)=1. \qquad (22)$$

The boundary conditions for a problem of heat transfer with uniform initial temperature distribution for a sphere are similar (8), (9).

Omitting any cumbersome transformations, we present down the solution in the image domain with respect to the complex variable *s*:

$$\theta(\bar{r},s)=\frac{1}{s}-\frac{Bi}{\bar{r}}\cdot\frac{sh\left(\sqrt{s}\bar{r}\right)}{s\cdot\varphi_2(s)}. \qquad (23)$$

Applying the inverse Laplace transform procedure [12] to the resulting expression, we obtain a solution in the area of originals for a sphere:

$$\theta(\bar{r},Fo)=\frac{2Bi}{\bar{r}}\sum_{n=1}^{\infty}\frac{\sin(\mu_n\bar{r})}{\mu_n(\mu_n\cdot\sin\mu_n-Bi\cdot\cos\mu_n)}\exp\left(-\mu_n^2Fo\right). \qquad (24)$$

SOLUTION FOR THE RANGE OF LOW FOURIER NUMBERS

In order to obtain a solution to the boundary value problems of heat and mass transfer for the range of low Fourier numbers, it is necessary to use the following properties of hyperbolic functions [15]:

$$ch\sqrt{s}=\frac{1}{2}\left(e^{\sqrt{s}}+e^{-\sqrt{s}}\right)\equiv\frac{1}{2}e^{\sqrt{s}};sh\sqrt{s}=\frac{1}{2}\left(e^{\sqrt{s}}-e^{-\sqrt{s}}\right)\equiv\frac{1}{2}e^{\sqrt{s}} \qquad (25)$$

$$ch\sqrt{s}\bar{x}=\frac{1}{2}\left(e^{\sqrt{s}\bar{x}}+e^{-\sqrt{s}\bar{x}}\right);sh\sqrt{s}\bar{x}=\frac{1}{2}\left(e^{\sqrt{s}\bar{x}}-e^{-\sqrt{s}\bar{x}}\right) \qquad (26)$$

As a result, expressions (11), (19), (23) are converted to the form:

- a plate (Fig. 1):

$$\theta(\bar{x},Fo)=\frac{1}{2\sqrt{\pi Fo}}\left\{\int_0^1\theta_0(\xi)\exp\left[-\frac{(\bar{x}\pm\xi)^2}{4Fo}\right]d\xi+\int_0^1\theta_0(\xi)\exp\left[-\frac{(2\pm\bar{x}-\xi)^2}{4Fo}\right]d\xi\right\}-$$

$$-Bi\exp(Bi^2Fo)\int_0^1\theta_0(\xi)\times\exp\left[Bi(2\pm\bar{x}-\xi)\right]erfc\left(\frac{2\pm\bar{x}-\xi}{2\sqrt{Fo}}+Bi\sqrt{Fo}\right)d\xi. \qquad (27)$$

- a cylinder (Fig. 3):

$$\theta(\bar{r},Fo)=\frac{1}{2\sqrt{\pi Fo\bar{r}}}\int_0^1\xi\frac{1}{2}\theta_0(\xi)\left\{\exp\left[-\frac{(\bar{r}-\xi)^2}{4Fo}\right]+\exp\left[-\frac{(2-\bar{r}-\xi)^2}{4Fo}\right]\right\}d\xi-$$

$$-\frac{Bi}{\bar{r}}\int_0^1\xi\frac{1}{2}\theta_0(\xi)\exp\left[Bi(2-\bar{r}-\xi)+Bi^2Fo\right]erfc\left[\frac{(2-\bar{r}-\xi)}{2\sqrt{Fo}}+Bi\sqrt{Fo}\right]d\xi \qquad (28)$$

- a sphere (Fig. 2):

$$\theta(\bar{r},Fo)=\frac{(\pm)1}{2\bar{r}\sqrt{\pi Fo}}\left\{\int_0^1\xi\theta_0(\xi)\exp\left[-\frac{(\bar{r}\pm\xi)^2}{4Fo}\right]+\int_0^1\xi\theta_0(\xi)\exp\left[-\frac{(2\pm\bar{r}-\xi)^2}{4Fo}\right]d\xi\right\}-$$

$$-(\pm)\frac{(Bi-1)}{\bar{r}}\exp\left[(Bi-1)^2Fo\right]\int_0^1\xi\theta_0(\xi)\exp\left[(Bi-1)(2\pm\bar{r}-\xi)\right]erfc\left[-\frac{2\pm\bar{r}-\xi}{2\sqrt{Fo}}+(Bi-1)\sqrt{Fo}\right]d\xi. \qquad (29)$$

The notation applies here:

$$(\pm)A(\mp B)=+A(-B)-A(+B). \qquad (30)$$

RESULTS AND DISCUSSION

As previously noted, the solutions to heat conduction and diffusion problems for solid bodies, including canonical ones, are obtained in the form of Fourier series [16-20], which is typical for the conditions with an uneven initial distribution of potentials of heat and mass transfer, but no solutions for the range of low Fourier numbers are given in appropriate sources. However, the shorter is the process time, the smaller are the numerical values of the Fourier criteria and thus there are more members of the infinite series, which leads to calculation error accumulation.

This paper presents solutions for bodies of canonical shape – a plate, a cylinder, and a sphere, and also it presents some nomograms of dimensionless body surface temperature depending on the values of the Biot and Fourier numbers for specific values of the *Bi* number. In this case, for simplicity, the calculations were performed for the conditions of a uniform initial distribution of the transfer potentials.

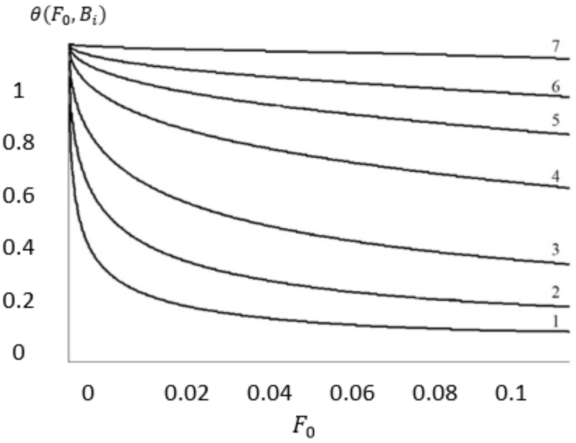

Fig. 1. Change in the dimensionless temperatures of the surface of a plate depending on the values of the Biot and Fourier numbers at Bi: 1) 20; 2) 10; 3) 5; 4) 2; 5) 1; 6) 0.5; 7) 0.1

Рис. 1. Изменение безразмерной температуры поверхности пластины в зависимости от значений чисел Био и Фурье при Bi: 1) 20; 2) 10; 3) 5; 4) 2; 5) 1; 6) 0,5; 7) 0,1

The nomograms allow us, using simple geometric operations (e.g., applying a ruler), to study the functional dependences of temperature on the surface of canonical bodies depending on the values of the Biot and Fourier numbers without performing any cumbersome calculations for low Fourier numbers, which contributes to the elimination of errors in the implementation of calculation methods using the «zonal» method [6-9] and the «micro-processes» method [3].

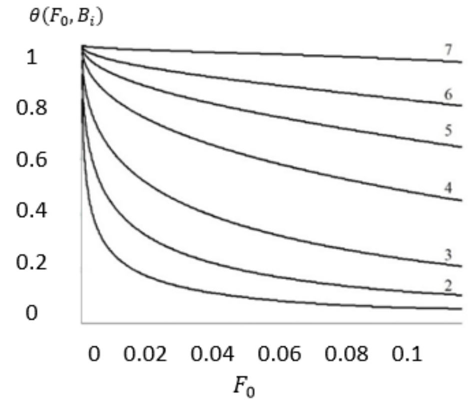

Fig. 2. Change in dimensionless temperatures of the surface of a ball depending on the Biot and Fourier numbers at Bi: 1) 20; 2) 10; 3) 5; 4) 2; 5) 1; 6) 0.5; 7) 0.1

Рис. 2. Изменение безразмерных температур поверхности шара в зависимости от чисел Био и Фурье при Bi: 1) 20; 2) 10; 3) 5; 4) 2; 5) 1; 6) 0,5; 7) 0,1

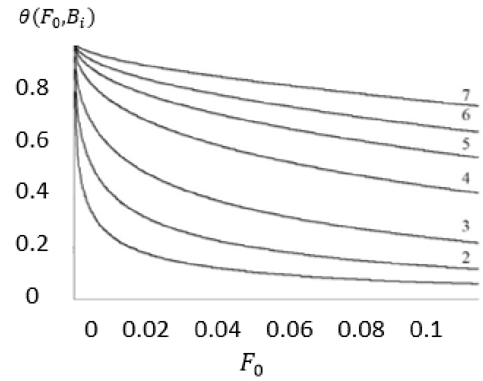

Fig. 3. Change in dimensionless temperature of the surface of a cylinder at Bi: 1) 20; 2) 10; 3) 5; 4) 2; 5) 1; 6) 0.5; 7) 0.1

Рис. 3. Изменение безразмерной температуры поверхности цилиндра при Bi: 1) 20; 2) 10; 3) 5; 4) 2; 5) 1; 6) 0,5; 7) 0,1

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