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МАТЕМАТИЧЕСКОЕ МОДЕЛИРОВАНИЕ ПРОЦЕССА СМЕШИВАНИЯ СЫПУЧИХ МАТЕРИАЛОВ В БАРАБАННО-ЛОПАСТНОМ СМЕСИТЕЛЕ

С.Н. Черпицкий, Л.В. Королев, М.Ю. Таршис

Сергей Николаевич Черпицкий (ORCID 0000-0001-6038-5238)*, Михаил Юльевич Таршис (ORCID 0000-0001-6398-7584)

Кафедра теоретической и прикладной механики, Институт машиностроения, Ярославский государственный технический университет, Московский пр-т, 88, Ярославль, Российская Федерация, 150023
E-mail: sergwinz@mail.ru*, mutarshis@yahoo.com

Леонид Владимирович Королев (ORCID 0000-0001-7980-3196)

Кафедра прикладной математики и вычислительной техники, Институт цифровых систем, Ярославский государственный технический университет, Московский пр-т, 88, Ярославль, Российская Федерация, 150023
E-mail: korolevlv@mail.ru

Предложена математическая модель процесса смешивания сыпучих материалов в аппарате гравитационно-пересыпного действия с дополнительными перемешивающими элементами. Смеситель включает в себя вращающийся вокруг горизонтальной оси цилиндрический корпус, внутри которого размещены ступени с рабочими лопастями и кольцевые перегородки, обеспечивающие как прямое движение материала вдоль оси вращения от загрузочного патрубка в приемный бункер, так и частичное перемещение материала в противоположном направлении, что позволяет увеличить время пребывания смеси внутри устройства. Для моделирования процесса смешивания использован метод дискретных элементов, в основе которого лежит представление сыпучего материала в виде совокупности представительных объемов, содержащих большое число частиц, взаимодействующих между собой и подвергающихся воздействию со стороны рабочих органов смесителя и внешних силовых полей. Размеры и массы представительных объемов пропорциональны размерам и массам частиц компонентов смеси. Их взаимодействие включает силу неупругого удара, направленного вдоль линии, соединяющей центры масс, и силу тангенциального трения, лежащую в плоскости, перпендикулярной этой линии, и направленную против проекции относительной скорости на плоскость. Абсолютные величины этих сил пропорциональны весу представительных объемов, экспоненциально убывают при удалении центров масс друг от друга на расстояние, превышающее сумму их размеров, и экспоненциально возрастают при сближении центров, вызывающих взаимное перекрытие представительных объемов. Аналогично моделируется и силовое взаимодействие представительных объемов с конструктивными элементами смесителя. Для проверки адекватности предложенной модели были проведены натурные эксперименты по смешиванию рапса и проса в изучаемом устройстве. Сопоставление результатов расчета по предложенной модели с данными экспериментальных исследований указывает на их удовлетворительную сходимость.

Ключевые слова: процесс смешивания, сыпучие материалы, моделирование, метод дискретных элементов

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MATHEMATICAL MODELING OF THE BULK MATERIALS MIXING PROCESS IN A DRUM-BLADE MIXER

S.N. Cherpitskiy, L.V. Korolev, M.Yu. Tarshis

Sergey N. Cherpitskiy (ORCID 0000-0001-6038-5238)*, Michael Yu. Tarshis (ORCID 0000-0001-6398-7584)

Department of Theoretical and Applied Mechanics, Institute of mechanical engineering, Yaroslavl State Technical University, Moskovskiy ave., 88, Yaroslavl, 150023, Russia
E-mail: sergwinz@mail.ru, mutarshis@yahoo.com

Leonid V. Korolev (ORCID 0000-0001-7980-3196)

Department Applied Mathematics and Computer Engineering, Institute of Digital Systems, Yaroslavl State Technical University, Moskovskiy ave., 88, Yaroslavl, 150023, Russia
E-mail: korolevlv@mail.ru

A mathematical model of the bulk materials mixing process in a gravity-flow apparatus with additional mixing elements is proposed. The mixer includes a cylindrical housing rotating around a horizontal axis, inside of which there are steps with working blades and annular partitions, which provide both direct movement of material along the axis of rotation from the loading nozzle to the receiving hopper, and partial movement of material in the opposite direction, which makes it possible to increase the time stay of the mixture inside the device. To simulate the mixing process, the method of discrete elements is used, which is based on the representation of a bulk material in the form of a set of representative volumes containing a large number of particles interacting with each other and exposed to the action of the working bodies of the mixer and external force fields. The representative volumes sizes and masses are proportional to the sizes and masses of the particles of the mixture components. Their interaction includes the force of inelastic impact directed along the line connecting their centers, and the force of tangential friction lying in the plane perpendicular to this line and directed against the relative velocity projection onto the plane. The absolute values of these forces are proportional to the weight of the representative volumes, exponentially decrease when the centers of mass are removed from each other by a distance exceeding the sum of their sizes, and exponentially increase when the centers approach, causing mutual overlap of the representative volumes. The force effect on the representative volumes when it comes into contact with the structural elements of the mixer is simulated in a similar way. To check the adequacy of the proposed model, full-scale experiments were carried out on mixing rapeseed and millet in the studied device. Comparison of the calculation results according to the proposed model with the data of experimental studies indicates their satisfactory convergence.

Key words: mixing process, bulk materials, modeling, method of discrete elements

INTRODUCTION

Among the devices for obtaining homogeneous bulk materials employed in various branches of chemical production and other industries, drum mixers with a rotating body have become widely used. This is due to the simplicity of their design and relatively low energy consumption. However, these devices are also characterized by low efficiency in the processing of components prone to segregation, in which denser and (or) smaller particles of the processed material are localized in the center of mixture circulation. The installation of additional working elements in the center of components circulation complicates the apparatus design, makes difficult its manufacture and maintenance. Nevertheless, in recent years, a number of devices [1, 2], the effectiveness of which is determined by using of elastic elements in the design, in particular, tires that worked out their resource, have been developed. In this case, additional working elements are made from the tire bead elements. Further improvement of this type apparatus design should be carried out on the basis of an adequate mathematical description of the mixing process [3].

To simulate the process of mixing bulk materials in gravity-flow devices, an approach based on representing the bulk mass as a continuous medium is usually used. It involves the formulation of two interrelated problems: determining the average velocity field of the material particles and finding the concentration field of the key component in the working volume, which is formed as a result of the movement of particles along the streamlines of the velocity field, as well as due to random displacements of particles from one streamline to another. To solve the first problem, an equations system for motion of a continuous medium is usually written down, which is solved by analytical or numerical methods, for example, the finite element method. To solve the second problem, kinetic equations in the discrete form of Markov chains [4–7] or equations of diffusion type in a continuous phase space [8-13] are often used. The application of the described approach encounters difficulties associated with the local nonequilibrium of granular media. The particles of the bulk medium in the devices operating in the collapse mode are not in an intense chaotic "temperature" movement. This leads to the fact that the fields of average velocity and concentration can undergo significant changes at distances comparable to the particle size. Such changes cannot be correctly described by the standard motion equations of a continuous medium and by kinetic equations of a diffusion type, which forces to make additional assumptions about the nature of the motion of a granular medium and the random processes occurring in it. In addition, solving the motion equations of a continuous medium for working volumes of a complex configuration with additional working elements that create discontinuities in the moving medium is not an easy task.

The discrete element method (DEM) seems to be more promising for modeling the mixing of bulk materials in the device under study [14-24]. The method is based on bulk material representation in the form of model particles set interacting with each other and exposed to the action of the working bodies of the device and external force fields. At the same time, the mass, size, elastic characteristics and interaction features of model particles are selected in such a way that they correspond to the properties of real particles as much as possible. To describe the particles force interaction, the model of elastic-viscous contact of Hertz–Mindlin solid spheres is usually used. The solution of motion equations system for model particles, taking into account all forces acting on them, makes it possible to obtain a picture of the spatial distribution of material fractions in the working volume of the device and the velocity field in it at each moment of time. The obvious disadvantage of this approach is the huge amount of calculations required to model systems containing tens of thousands or more particles. In addition, in many applications, a detailed description of the system at the level of individual particles is redundant. For example, to assess the quality of a mixture, it is sufficient to know the particle distribution of a key component averaged over the volume of a sample cell containing a large number of particles. In [20-21], the authors combine several (from 2 to 10) solid spherical particles (corresponding to real particles of bulk material) into one "coarse" particle ("coarse grain") to reduce the amount of calculations. Despite the fact that the particles that make up the "large" particles are not rigidly connected to each other, the authors consider the interaction of "large" particles elastic-viscous.

THE MIXER DESIGN SCHEME DESCRIPTION AND THE METHOD OF MODELING THE MIXING PROCESS

The design diagram of the mixer [1] is shown in Fig. 1. It includes a cylindrical housing 1, inside which there are stages 2 with working blades, transparent annular partitions 3 and transparent end walls – rings 4. The components are loaded through the nozzle, and unloading through the hole in the ring 4. The mixer drive is not shown. The mixing components through the hole in the left end wall 4 enter the housing 1. After turning on the drive, the housing 1 with stages 2 and blades begins to rotate. In this case, the components inside the body move in a roll-over mode, that is, they rise and fall. The mixing of the components occurs mainly in the flow of the collapse of the material, as well as when the mixture is shuffled by the blades. The blades of each stage 2 are bent alternately, in opposite directions, due to which the circulation of bulk material is ensured not only in the cross section of the mixer, but also in its longitudinal section.

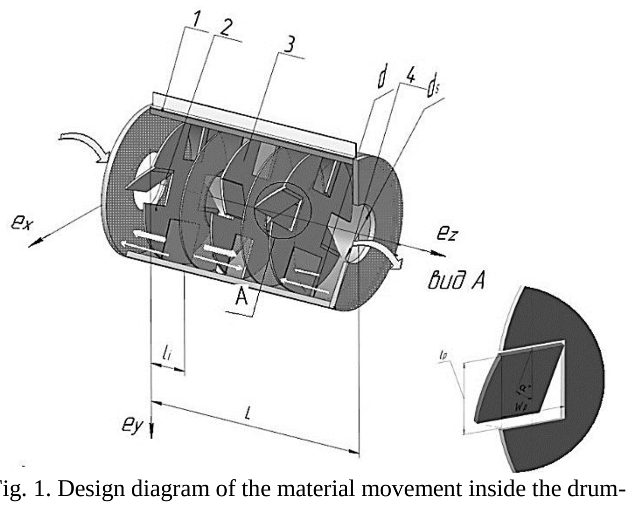

Fig. 1. Design diagram of the material movement inside the drum-blade mixer

Рис. 1. Расчетная схема движения материала внутри барабанно-лопастного смесителя

In this paper, the following DEM implementation method is proposed for modeling a bulk medium. We have divided the entire volume of bulk material into representative volumes of size and mass, each of which contains the same number of particles of one of the components to be mixed. Thus, the size and mass ratios of representative volumes are equal, respectively, to the size and mass ratios of the particles filling them. The size of the representative volumes may vary, but it should be small both in comparison with the characteristic dimensions of the working volume and in comparison with the size of the test cell into which the volume of the mixture under study is divided when calculating the inhomogeneity coefficient. We consider these representative volumes as model particles for which we write down the equations of motion. Although the shape of these particles, generally speaking, remains uncertain and may change during the process, we will conditionally represent these particles in the form of balls with radii R_i. The position of the center of the particle and its velocity at each moment of time in the adopted frame of reference are set, respectively, by vectors $\vec{r}_i$ and $\vec{v}_i$ ($i=1..N$).

We will assume that the model particles have no rotational degrees of freedom. As it noted in [23], DEM models, that do not take into account the rotational degrees of particles freedom, adequately reflect the bulk material movement. Then the particle motion equations system will have the form [23]:

$$\begin{cases} \dfrac{d\vec{r}_i}{dt} = \vec{v}_i \\ \dfrac{d\vec{v}_i}{dt} = \vec{g} + \dfrac{1}{m_i}\left(\sum_j \vec{F}_{ij} + \sum_k \vec{P}_{ik}\right) \end{cases} \quad i=1..N \tag{1}$$

Where $\vec{g}$ – acceleration of gravity, $\vec{F}_{ij}$ – the force with which the numbered particle j acts on the particle with the number i, $\vec{P}_{ik}$ – the force with which on a particle with a number i acts the device element with a number k. As the device elements, we will consider the cylindrical housing 1 of the mixer and flat internal devices: blades 2 and sectional partitions 3.

We represent the force of interaction of particles as the sum of two terms (Fig. 2): force of inelastic repulsion $\vec{F}^{\,r}_{ij}$, acting along the vector $\Delta\vec{r}_{ij} = \vec{r}_i - \vec{r}_j$, connecting the centers of particles, and the friction force $\vec{F}^{\,p}_{ij}$, acting against the component of relative velocity vector $\Delta\vec{v}_{ij} = \vec{v}_i - \vec{v}_j$ of the particles, lying in a plane perpendicular to the vector $\Delta\vec{r}_{ij}$:

$$\vec{F}_{ij} = \vec{F}^{\,r}_{ij} + \vec{F}^{\,p}_{ij}, \tag{2}$$

$$\vec{F}^{\,r}_{ij} = |\vec{g}|\, m_{ij} A(i,j)\, \vec{n}_{ij}\left(1 - k_r(\vec{n}_{ij}\vec{\eta}_{ij})\,\theta\big((\vec{n}_{ij}\vec{\eta}_{ij}) < 0\big)\right), \tag{3}$$

$$\vec{F}^{\,p}_{ij} = -|\vec{g}|\, m_{ij} A(i,j)\, k_p\left(\vec{\eta}_{ij} - \vec{n}_{ij}(\vec{n}_{ij}\vec{\eta}_{ij})\right), \tag{4}$$

where $A(i,j)$ is defined by the following formula

$$A(i,j) = \exp\left(-\frac{|\Delta\vec{r}_{ij}| - (R_i + R_j)}{k_d R_{ij}}\right). \tag{5}$$

Symbols m_{ij} and R_{ij} denote, respectively, the transformed mass and the transformed radius of the particles.

$$m_{ij} = \frac{m_i m_j}{m_i + m_j}, \quad R_{ij} = \frac{R_i R_j}{R_i + R_j}, \tag{6}$$

$\vec{n}_{ij}$ and $\vec{\eta}_{ij}$ - unit vectors of relative position and relative velocity of particles

$$\vec{n}_{ij} = \frac{\Delta\vec{r}_{ij}}{|\Delta\vec{r}_{ij}|}, \quad \vec{\eta}_{ij} = \frac{\Delta\vec{v}_{ij}}{|\Delta\vec{v}_{ij}|}. \tag{7}$$

k_d, k_r, k_p – dimensionless positive coefficients.

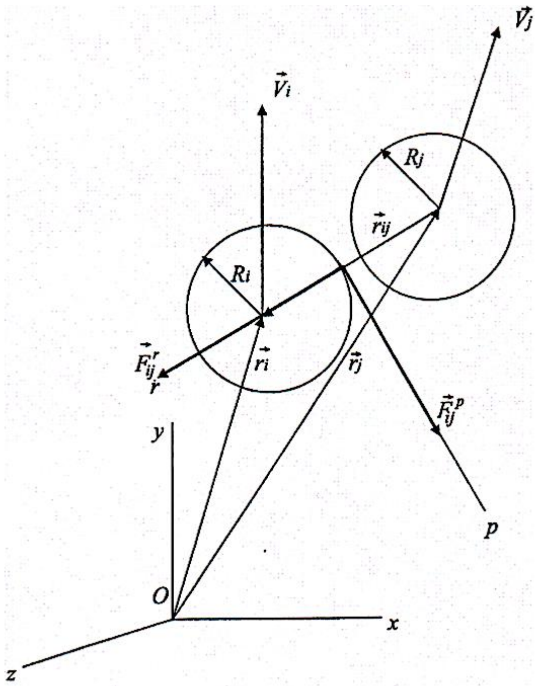

Fig. 2. Particle interaction scheme
Рис. 2. Схема взаимодействия частиц

Function $\theta(z)$ defined by the expression:

$$\theta(z) = \begin{cases} 0, & z = False, \\ 1, & z = True, \end{cases} \tag{8}$$

From expressions (2) - (8) it is seen: $\vec{F}_{ij} = -\vec{F}_{ji}$, that is, the forces with which two arbitrarily chosen particles act on each other are equal in magnitude and opposite in direction.

The proposed model of particles interaction (2)-(8) differs from the one adopted in most works on a grain medium modeling by the DEM method [14-23]. This is due to the fact that within the framework of our approach, each model particle represents a large number of unrelated real particles of bulk material, and therefore should be considered plastic rather than elastically viscous. The characteristic scale of the interaction forces of particles at the point of contact $|\Delta\vec{r}_{ij}| = (R_i + R_j)$ is determined not by their elastic modules, but by the product of the acceleration modulus of gravity by the reduced mass m_{ij}. Such a choice seems to be justified for modeling the movement of bulk material in a gravity-flow device, when the main factor causing the interaction of particles is the gravity field.

Change in the absolute values of the acting forces with distance $|\Delta\vec{r}_{ij}|$ between the centers of particles is determined by the dimensionless amplitude $A(i, j)$ (5), which is expressed by an exponential function. The choice of such a smooth dependence, in contrast to the power function, adopted in the Hertz–Mindlin model, is also due to the plasticity of model particles. The dependence of the amplitude $A(i, j)$ on the distance $|\Delta\vec{r}_{ij}|$ is schematically shown in Fig. 3.

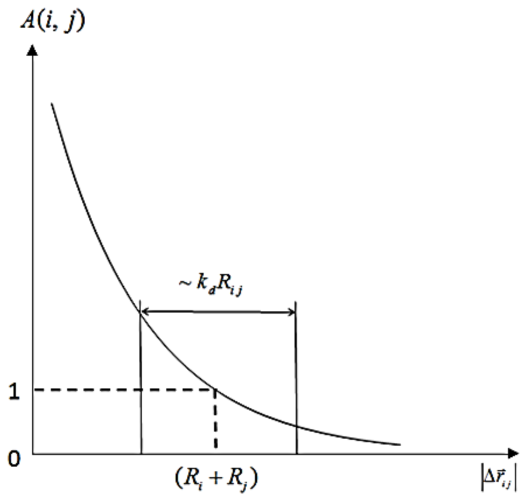

Fig. 3. Change in the absolute values of forces with the distance between particles

Рис. 3. Изменение абсолютных величин сил с расстоянием между частицами

At $|\Delta\vec{r}_{ij}| = (R_i + R_j)$ the amplitude takes on the value 1, and scale of a distance $|\Delta\vec{r}_{ij}|$, at which it changes significantly is equal to $k_d R_{ij}$. The intensity of various types of particle interactions is determined by the coefficients k_d, k_r, k_p, included in expressions (3)-(5).

The coefficient k_d establishes, divided into the transformed radius of the particles R_{ij}, the size of the region in the vicinity of the particles point of contact, in which their interaction takes place. At $k_d \to 0$ particles become absolutely "solid", the force of mutual repulsion increases sharply from 0 at $|\Delta\vec{r}_{ij}| > (R_i + R_j)$ ad infinitum at $|\Delta\vec{r}_{ij}| < (R_i + R_j)$. At large values k_d, the "hardness" of the particles decreases, the forces in the vicinity of the contact point change more smoothly. Coefficient k_r determines the degree of "inelasticity" of the repulsive force, that is, the difference between the repulsive forces acting upon mutual approach and removal of particles. Such a feature of the interaction is characteristic of ductile particles [24]. At $k_r \to 0$ repulsion becomes elastic, with increasing k_r the loss of kinetic energy of particles in collisions increases.

The magnitude of the friction force arising when one particle slides over the surface of another is proportional to the magnitude of the repulsive force arising between them $A(i, j)$ and the coefficient of friction k_p.

The particle is also acted upon by the forces of inelastic repulsion and friction from the structural elements of the mixer (cylindrical housing 1, blades 2 and partitions 3). Since these forces are short-range, and the radii of the model particles are much smaller than the dimensions of all elements of the device, it can be assumed that the interaction of the i-th particle with the k-th element of the device occurs in a small neighborhood of the point $\vec{r}_{ik}^{(u)}$, lying on the surface of an element at the shortest distance from the center of the particle $\vec{r}_i$. When rotating the mixer with an angular velocity $\vec{\omega}$, the speed of the contact point is determined by the vector product

$$\vec{v}_{ik}^{(u)} = [\vec{\omega}\vec{r}_{ik}^{(u)}]. \qquad (9)$$

Then the force $\vec{P}_{ik}$, with which the element k of the mixer acts on the particle i can, based on expressions (2)-(5), be represented as the sum of inelastic repulsive forces $\vec{P}_{ik}^{r}$ and friction $\vec{P}_{ik}^{p}$:

$$\vec{P}_{ik} = \vec{P}^{r}_{ik} + \vec{P}^{p}_{ik}, \quad (10)$$

$$\vec{P}^{r}_{ik} = |\vec{g}| m_i B(i,k) \vec{n}^{(u)}_{ik} \left(1 - k^{(u)}_r (\vec{n}^{(u)}_{ik} \vec{\eta}^{(u)}_{ik}) \theta\left((\vec{n}^{(u)}_{ik} \vec{\eta}^{(u)}_{ik}) < 0\right)\right), \quad (11)$$

$$\vec{P}^{p}_{ik} = -|\vec{g}| m_i B(i,k) k^{(u)}_p \left(\vec{\eta}^{(u)}_{ik} - \vec{n}^{(u)}_{ik} (\vec{n}^{(u)}_{ik} \vec{\eta}^{(u)}_{ik})\right), \quad (12)$$

where

$$B(i,k) = \exp\left(-\frac{\left|\Delta \vec{r}^{(u)}_{ik}\right| - R_i}{k^{(u)}_d R_i}\right) \quad (13)$$

and

$$\vec{n}^{(u)}_{ik} = \frac{\Delta \vec{r}^{(u)}_{ik}}{\left|\Delta \vec{r}^{(u)}_{ik}\right|}, \quad \vec{\eta}^{(u)}_{ik} = \frac{\Delta \vec{v}^{(u)}_{ik}}{\left|\Delta \vec{v}^{(u)}_{ik}\right|},$$

$$\Delta \vec{r}^{(u)}_{ik} = \vec{r}_i - \vec{r}^{(u)}_{ik}, \quad \Delta \vec{v}^{(u)}_{ik} = \vec{v}_i - \vec{v}^{(u)}_{ik} \quad (14)$$

Dimensionless positive coefficients $k^{(u)}_d$, $k^{(u)}_r$, $k^{(u)}_p$ have the same meaning as the coefficients k_d, k_r, k_p in expressions (2) - (5).

To find the forces $\vec{P}_{ik}$ by formulas (10) - (14), it is necessary to have expressions for the points of contact i-th particles c k-th mixer element $\vec{r}^{(u)}_{ik}$, which depend on the shape of the elements and the layout of the mixer and are determined using elementary geometric calculations.

To solve system (1), taking into account expressions (2) – (14), it is necessary to set the initial velocities $\vec{v}_i(t=0)$ and starting positions $\vec{r}_i(t=0)$ model particles. In this work, all $\vec{v}_i(t=0)$ were assumed equal to 0. The initial positions of the particles should be determined in accordance with the given initial loading of the mixed fractions, so that the spherical volumes occupied by the particles do not overlap with each other and with the installation elements.

COMPARISON OF THE NUMERICAL CALCULATIONS RESULTS WITH EXPERIMENTAL DATA

The system of ordinary differential equations (1)-(14) was solved numerically. For this purpose, an algorithm has been developed, which was implemented using the software tools of the Mathematica environment. The calculated spatial distribution of model particles corresponding to the key and transporting components of the mixture was used to determine the inhomogeneity coefficient V_C of the mixture.

The values of the inhomogeneity coefficient in the section (layer) at a distance L from the entry point of the material into the mixer were calculated by the formula:

$$V_C = \frac{1}{\bar{c}} \sqrt{\frac{1}{\nu} \sum_{i=1}^{n} \nu_i (c_i - \bar{c})^2}, \quad (15)$$

where $\bar{c}$ – the volume-average concentration of the key component in a transverse layer of bulk material with a thickness equal to the diameter of the largest particle, ν – the volume of particles trapped in the layer, c_i – volumetric concentration of the key component in i-th sample – subdomains of the layer, ν_i – volume of particles in i-th sample.

To check the proposed model adequacy, the calculation results were compared with the natural experiments on mixing rapeseed and millet data [1, 2]. Investigations of the mixing process were carried out by analyzing images of cross-sections of the mixture [2], that fixed through transparent end walls 4 and annular partitions 3, which were installed in the certain mixer cross sections at a given distance (L) from the place of components supply (left wall 4). Comparison of the calculation results with the data of field experiments is shown in Fig. 4. Numerical and field experiments were carried out with the following mixer parameters. The angle of the blades inclination to the housing cross-section plane $\alpha_1 = 60^\circ$, $\alpha_2 = 30^\circ$. Width w and length l of blades: 1 – w = 60 mm, l = 60 mm (all steps); 2 – w = 60 mm, l = 60 mm (1 and 2 steps); w = 60 mm, l = 20 mm (stage 3); 3 – w = 60 mm, l = 60 mm (1 step); w = 60 mm, l = 20 mm (2 and 3 steps). The ratio of the density and particle size of the key and carrier components was 0.786 and 0.778, respectively, the volume fraction of the key component was 0.33, and the fill factor was-0.21. Numerical calculation with the number of model particles $N = 480$ was executed with parameters: $k_d = 0.25$, $k_r = 15$, $k_p = 0.15$, $k^{(u)}_d = 0.2$, $k^{(u)}_r = 10$, $k^{(u)}_p = 0.1$. These values were determined during numerical experiments out of the qualitative requirements correspondence of the calculated pattern of bulk material movement to the movement observed in a full-scale experiment, and the quantitative correspondence of the spatial distribution of the model particles of the key and transporting components to the distribution of the corresponding particles in the real mixture according to its value of the inhomogeneity coefficient.

The Fig. 4 shows the graphs of mixture heterogeneity coefficient behaviors as a function of the cross section distance L from the place of the components loading.

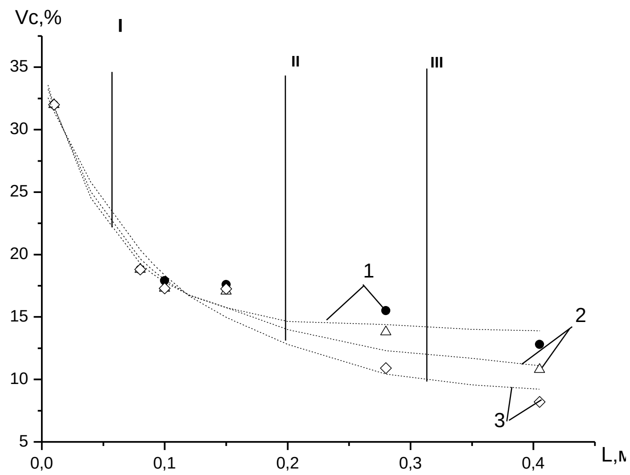

Fig. 4. Coefficient of the mixture heterogeneity at a distance L from the place of its loading with different designs of blades (2 and 3 steps) of the three-stage mixer (rapeseed - millet) experiment (symbols) and calculation (dashed line). I, II, III - blade stages installation locations

Рис. 4. Коэффициент неоднородности смеси на расстоянии L от места её загрузки при различных конструкциях лопастей (2 и 3 ступеней) трёхступенчатого смесителя (рапс - просо) эксперимент (символы) и расчет (пунктир). I, II, III - места установки лопастных ступеней

Comparison of the experimental data and the results of numerical calculations demonstrate their satisfactory overlap. Thus, the considered mathematical model, based on the processed material representation in the form of a set of model particles interacting with each other and the elements of the mixer, makes it possible to give an adequate description of the mixing process in continuous devices and can be used as the basis for the engineering method of their calculation.

DISCUSSION

The paper proposes the variant of DEM method of a bulk material movement modeling in a drum-blade mixer. Unlike the known variants of the implementation of this method [14-24], we take as model particles not individual particles of bulk material, but representative volumes containing a large number (hundreds) of individual particles, which significantly reduces the amount of calculations. Since the grains of the bulk material that make up the model particle are not rigidly interconnected, this particle exhibits plastic rather than elastic properties, which leads to the need for significant changes in the model of force interaction of model particles with each other and with parts of the mixer compared to the model of elastic-viscous contact of hard Hertz–Mindlin spheres adopted in most works. The proposed model is universal in the sense that it does not contain parameters characterizing the rheological properties of the particle material of a particular bulk medium and the geometry of these particles. Only two parameters of the model k_p and $k_p^{(u)}$ are significantly related to the nature of the bulk material. They are proportional, respectively, to the coefficient of internal friction of the bulk mass and the coefficient of friction in the system "bulk mass – mixer housing material". Such a model, in our opinion, corresponds to a picture of the movement of various bulk media in devices of gravity-bulk action.

The authors declare the absence a conflict of interest warranting disclosure in this article.

Авторы заявляют об отсутствии конфликта интересов, требующего раскрытия в данной статье.

ЛИТЕРАТУРА

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