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ПОСТРОЕНИЕ И ВЕРИФИКАЦИЯ МОДЕЛИ ФОРМИРОВАНИЯ СТРУКТУРЫ И ТЕПЛООБМЕНА В АППАРАТЕ КИПЯЩЕГО СЛОЯ С ТЕПЛОВОЙ РУБАШКОЙ

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В работе на основе математического аппарата теории цепей Маркова предложена одномерная математическая модель формирования псевдоожиженного слоя и эволюции его теплового состояния в процессе прогрева ансамбля частиц. При описании теплового состояния системы учитываются процессы межфазного теплообмена между газовым потоком и частицами, а также между стенкой аппарата и газовзвесью. Результаты выполненных ранее исследований и известные эмпирические закономерности для материальных констант процесса использованы для идентификации параметров модели. Выполненная параметрическая идентификация позволила адаптировать и применить математическую модель для расчета процесса тепловой обработки навески частиц силикатного песка в аппарате с псевдоожиженным слоем периодического принципа действия, снабженного рубашкой нагрева. Проверка прогнозирующих возможностей адаптированной таким образом физико-математической модели была проведена путем сравнения получаемых расчетным путем прогнозов с результатами выполненного в ходе исследования натурного эксперимента. Для проведения натурного эксперимента навеска частиц песка помещалась в аппарат, который предварительно выводился на установившийся тепловой режим работы. После загрузки материала в аппарат помещали набор термопар, равноудаленных друг от друга, и снимали показания температуры на различных высотах от уровня газораспределительной решетки. Полученные таким образом распределения температур по высоте слоя находились в хорошем соответствии с расчетными прогнозами и показали значительную неоднородность полей температуры газовой фазы. Температура частиц оценивалась только как интегральная характеристика по измерениям теплосодержания сыпучей среды после выгрузки ее навески из аппарата. Интегральная температура частиц также оказалась близка к прогнозным значениям. Кроме того, в ходе расчетно-экспериментального исследования были уточнены кинетические характеристики нагрева навески материала. Таким образом, предложенная математическая модель обладает достаточной для инженерных задач прогностической эффективностью и может рассматриваться в качестве основы для построения компьютерного метода расчета теплообменных аппаратов с рубашкой нагрева, использующих технику псевдоожижения сыпучих сред.

Ключевые слова: псевдоожиженный слой, рубашка нагрева, теплообмен, численное моделирование, цепи Маркова

DESIGN AND VERIFICATION OF THE MODEL OF STRUCTURE FORMATION AND HEAT TRANSFER IN A FLUIDIZED BED APPARATUS WITH A HEAT JACKET

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A one-dimensional mathematical model of the formation of a fluidized bed and the evolution of its thermal state during the heating of an ensemble of particles is proposed based on the mathematical apparatus of the Markov chain theory. When describing the thermal state of the system, the processes of interfacial heat exchange between the gas flow and particles, as well as between the wall of the apparatus and the gas suspension, were taken into account. The results of previous studies and known empirical regularities for the material constants of the process were used to identify the parameters of the model. The parametric identification made it possible to adapt and apply the mathematical model for calculating the process of heat treatment of the suspension of silicate sand particles in an apparatus with a fluidized bed of a periodic principle of operation equipped with a heating jacket. Verification of the predictive capabilities of the physicomathematical model adapted in this way was carried out by comparing the forecasts obtained by calculation with the results of the full-scale experiment conducted during the study. To conduct the full-scale experiment, a sample of sand particles was placed in the apparatus, which was previously output to the established thermal regime of operation. After loading the material, a set of thermocouples equidistant from each other was placed in the apparatus and temperature readings were taken at different heights from the level of the gas distributor. The temperature distributions obtained in this way along the bed height were in good agreement with the calculated forecasts and show a significant heterogeneity of the gas phase temperature fields. The temperature of the particles was estimated only as an integral characteristic by measurements of the heat content of the bulk medium after unloading from the apparatus. The integral temperature of the particles also turned out to be close to the predicted values. In addition, during the computational and experimental study, the kinetic characteristics of the heating of the suspension of the material were clarified. Thus, the proposed mathematical model has sufficient predictive efficiency for engineering tasks and can be considered as a basis for constructing a computer method for calculating heat exchangers with a heating jacket using the fluidization technique of bulk media.

Key words: fluidized bed, heating jacket, heat transfer, numerical modeling, Markov chains

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INTRODUCTION

Fluidized bed apparatuses are widely used in chemical [1-2], energy [3-4], pharmaceutical [5-6], food [7-8] and other fields of material production. With the expansion of the options for the practical use of the technique of fluidization of granular materials and the accumulation of theoretical and experimental results of studies of various aspects of fluidized beds, the task of engineering and technical workers, in our opinion, did not turn easier because it became almost impossible for one person to perceive all the available information about the results of research [9].

The second factor, which also complicates the calculation of devices with a fluidized bed, is that as even individual characteristics of fluidized systems are studied (for example, the parameters of phase migration in the bed), the accumulated knowledge about the subject of research makes the calculation methods too detailed, which ultimately leads to the formation of stable approaches for describing processes in the fluidized bed, but makes it difficult to quantify models parameters. Without doing a detailed analysis and classification of possible approaches to the description of fluidized bed processes in this paper, we will give only a few concrete examples that characterize, in our opinion, the current state of approaches to the calculation of fluidized bed apparatuses.

The so-called two-phase fluidization theory is widely used in the implementation of applied calculations, the development of which approach was apparently initiated by the work of Toomey and Johnston [10]. The development of this theory has formed a stable approach for modeling the fluidized bed as a result of the mechanical combination of a discrete (gas) phase in the form of ascending gas bubbles formed from gas jets, as well as bulk material, which is considered as a continuous medium [9-12]. Despite the fact that such representations are quite artificial, they have shown their productivity within the framework of many engineering calculation methods [9].

At the same time, such fluidization theories were also developing, which were the result of the use of mathematical tools already widely used in other subject areas. Thus, when considering particles and a fluidizing agent as interpenetrating continuous media, it is logical to use the equations of hydromechanics with their initial and boundary conditions to describe them (ultimately, this led to the appearance and use of computational fluid dynamics software packages) [13-14]. Also, when describing the movement of grains of bulk media in a fluidized bed, the discrete element method (DEM), also called the discrete particle method (DPM), was used [15-17]. Such models are based on using of the conservation laws for angular momentum and quantity of motion for an individual particle, while the carrier medium is often described based on the equations of hydromechanics [17]. In our opinion, the considered approaches occupy a dominant position in the study of the motion of phases of fluidized beds, although, of course, it is not difficult to give examples of solving such problems using other mathematical tools (for example, Markov chain theory [19], the mathematical apparatus of the kinetic theory of gases [20], lattice Boltzmann equations [21], etc.).

Summing up, it can be mentioned that even a brief analysis of the development and current state of the issue of modeling the operation of apparatuses with a fluidized bed allows us to make the fairly unambiguous conclusions. Firstly, the fluidized bed of particles is a complex system, even the qualitative characteristics of which can vary greatly depending on the properties of the particles, on the mode of motion of the fluid medium, on the design of the apparatus, etc. [8-9]. Secondly, despite the fact that numerous approaches have been proposed and developed to date to describe the fluidized bed as an object with distributed characteristics, predictively effective models are being developed for a specific situation and require empirical refinement, and the most detailed models require identification of a significant number of parameters, which greatly complicates their use. As a result, when studying the process in a specific apparatus with a fluidized bed, it is necessary to choose an approach to modeling, taking into account the feasibility of identifying the parameters of all interrelated submodels of elementary processes.

MATERIALS AND METHODS

Theoretical foundations of physical and mathematical modeling

The mathematical model proposed in this study is based on the consideration of the flow of physico-chemical and transport processes in a fluidized bed using the mathematical apparatus of Markov chains with discrete time, which have shown their productivity for the processing of bulk materials [22-23].

The object of the research in this work is a fluidized bed installation with a heating jacket. Such devices are quite widespread in industrial practice [24-26]. The most similar in its design scheme to the mathematical model formed in this paper is the previously developed fluidized bed model with time-varying properties of particles [19]. The basic assumptions used earlier [19] to describe the formation of a fluidized bed and gas-particle heat exchange are generally preserved in this study.

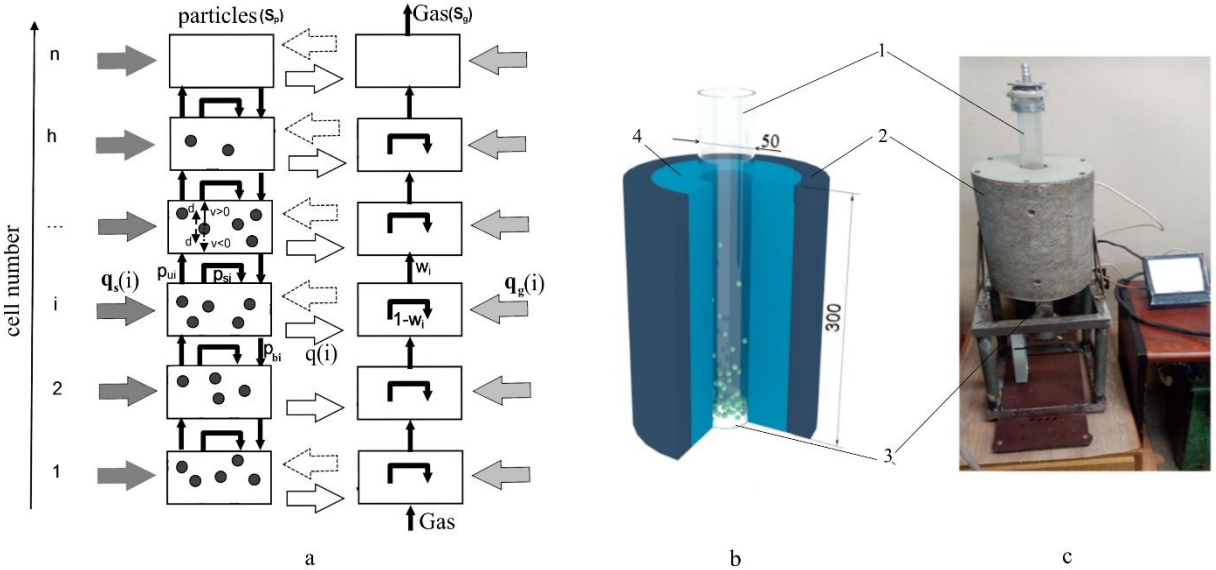
At the same time, of course, a revision of the parametric identification procedures is required, in addition, the bed-wall heat exchange was not considered in principle in the previous work [19] and is being introduced for the first time in this study. In this regard, the main attention in this paper will be paid to the description of the «external» bed-wall heat transfer, and the description of the formation of the fluidized bed and the «internal» heat exchange will be described poorly (the details can be found in [19]).

The design scheme of the proposed mathematical model is shown in Fig. 1a, the scheme of the main part of the apparatus is shown in Figure 1b, and a photograph of the appearance of the laboratory setup is shown in Fig. 1c.

It is known that particle motion has an intensifying effect on heat exchange with the surface due to the reduction of the boundary layer and the maintenance of high temperature drop values, however, no generally accepted model of the heat exchange mechanism has been proposed [24-25]. Separate consideration of heat fluxes associated with particles and with the gas phase does not give advantages in terms of explaining the mechanism of heat transfer between wall and suspension, therefore, bed-wall heat exchange is considered in practice [24]. Thus, the bed-wall heat transfer is usually described taking into account the average temperature drop unchanged along the length of the heat exchange surface [9, 24].

In this paper, the space of the apparatus is considered as a set of a countable number of n representative volumes (Markov chain cells), which makes it possible to describe the bed as an object with height-distributed parameters.

Any cell is characterized by a small finite size in height Δx to build a one-dimensional model of the process. The state of the object is characterized by sets of parameters organized into state vectors **S** (to characterize the distribution of the masses of the gas and solid phases along the corresponding chains) and **Q** (to characterize the distribution of heat along the corresponding chains). The evolution of the state vectors is calculated at discrete time points $t_k = (k - 1)\,\Delta t$, where Δt is the time step (the time interval between the nearest states of the object); k is the time step number (is an integer analog of time). The temperature of the wall T_w is assumed to be constant, so the temperature pressure is completely determined by the variable temperatures of particles and gas in the height of the bed, the content of which also varies depending on the axial coordinate.



a b c

Fig. 1.a – calculation scheme of modeling the phases movement and heat transfer in a cylindrical fluidized bed apparatus with a heating jacket; б – scheme of the apparatus (1 – cylindrical apparatus; 2 – thermal insulation; 3 – gas distributor; 4 – heater); в – appearance of the apparatus (1 – cylindrical apparatus; 2 – heater; 3 – gas distributor)

Рис. 1.а – расчетная схема моделирования движения фаз и переноса теплоты в цилиндрическом аппарате кипящего слоя с рубашкой нагрева; б – схема аппарата (1 – цилиндрический аппарат; 2 – нагреватель; 3 – газораспределительная решетка; 4 – теплоизоляция); в – внешний вид аппарата (1 – цилиндрический аппарат; 2 – нагреватель; 3 – газораспределительная решетка)

The evolution of the object from the considered state to the next is described by multiplying the state vector by the corresponding transition matrix **P** (with the indexes "p" and "g" for the solid and gas phases, respectively), which is the main operator of models based on the theory of Markov chains [19, 22].

Thus, the migration of particles and the advance of the gas phase along the apparatus is described by recurrent matrix equalities [19, 26]:

$$\mathbf{S}_p^{k+1} = \mathbf{P}_p^k \mathbf{S}_p^k; \quad (1)$$

$$\mathbf{S}_g^{k+1} = \mathbf{P}_g^k \mathbf{S}_g^k + \mathbf{S}_{gf}, \quad (2)$$

where $\mathbf{S}_p$ and $\mathbf{S}_g$ are the column vectors characterizing the distribution of solid and gas phases; $\mathbf{P}_p^k$ and $\mathbf{P}_g^k$ are the transition probability matrices for the corresponding phases; $\mathbf{S}_{gf}$ is the source vector of the gas flow (has a single non–zero element in the first cell equal to the volume of gas supplied under the gas distributor in one time interval Δt).

The calculation scheme of the mathematical model assumes that the local velocity of the gas phase in the cell is calculated taking into account the reduction in the cross section of the gas flow due to the presence of particles. Taking into account the constrained nature of the gas movement leads to higher values of the defined heat exchange criteria. At the same time, the volume content of the solid phase in the cell is formed taking into account the local velocity, which makes the proposed model nonlinear [19]. The relationship between the concentration of particles and the gas velocity is given by the ratio [19]:

$$w = \frac{W_0}{1-\pi\left(S/8 \cdot S_{max}\right)^{2/3}}, \quad (3)$$

where S and S_{max} are the corresponding (k–th) moment of time and the maximum (for a dense bed) values of the particle content in the cell (the index of the cell number is not given here).

The probabilities of particle migration from the i-th cell forming the i-th column of the $\mathbf{P}_p$ transition matrix are related to the process parameters by the following dependencies [19]:

$$p_{si} = 1 - p_{ui} - p_{di}; \quad (4)$$

$$p_{di} = d_i \text{ for } (w_i - V_{si}) > 0; \quad (5)$$

$$p_{di} = v_i + d_i \text{ for } (w_i - V_{si}) < 0; \quad (6)$$

$$p_{ui} = v_i + d_i \text{ for } (w_i - V_{si}) > 0; \quad (7)$$

$$p_{ui} = d_i \text{ for } (w_i - V_{si}) < 0, \quad (8)$$

where d is the probability of diffusion transfer (a symmetric component the total probability of migration), introduced to account for random factors determining particle motion and associated with the macrodiffusion coefficient D through the ratio [19, 26]:

$$d = D \cdot \Delta t / \Delta x^2; \quad (9)$$

where v is the convective probability of particle migration from the cell, determined by the local velocity of the gas w_i and the aerodynamic size of the particle (characterized by the settling velocity of the particle V_s):

$$v = |w_i - V_{si}| \cdot \Delta t / \Delta x. \quad (10)$$

The system of equations (1)-(10) allows us to describe the formation of distributions of particles and gas along the height of the apparatus (along the corresponding chains), that is, ultimately describe the intensity of the transfer of extensive properties in the space of the apparatus. This information is the primary basis for describing heat transfer and the formation of temperature distributions, taking into account phase migrations, interphase heat exchange and the presence of external sources.

The velocity of the constrained gas movement (Eq. (3)) largely determines the probabilities of particle migrations, while at the same time it depends on the presence of particles in the cells, which makes the proposed model nonlinear. The amount of heat q_i transferred between particles and gas in the i-th cell in one time step also depends on the rate of gas movement, the interfacial surface **F**, which depends on the content of particles in the cell. As a result, the intensity of interphase interaction is determined in the usual way [9], but is calculated from local state parameters and is calculated for the i-th cell as [19]:

$$q_i^k = \alpha_i^k \cdot F_i^k \cdot \Delta t \cdot (T_{p,i}^k - T_{g,i}^k), \quad (11)$$

where α is the gas-solid heat transfer coefficient, F is the total heat exchange surface, which depends on the concentration of the material in the cell, T_p and T_g are the temperatures of the solid and gas phases.

It is difficult to separate the calculation of the heat flow from the wall to the particles (q_s) and from the wall to the gas phase (q_g), therefore, only the total heat flow (q_w) from the wall is usually subject to calculation [9, 24-25]. In other words, the bed is considered as some medium with a temperature T_s, flowing around the heat exchange surface F_w, and the amount of heat entering the i-th cell can be written as follows:

$$q_{w,i}^k = \alpha_{w,i}^k \cdot F_{w,i}^k \cdot \Delta t \cdot (T_w^k - T_{s,i}^k), \quad (12)$$

where α_w is the wall-bed heat transfer coefficient, F_w is the exchange surface, T_w and T_s are the wall and gas suspension temperatures. The suspension temperature it is taken as the average between the temperature of the gas and the particles, and the wall temperature is assumed to be constant (T_w = 925 °C).

Since the heat coming from the wall to the gas suspension has to be redistributed in some way between the gas and solid phases, the assumption was made in the framework of constructing the model that heat is transferred to particles in portions depending on porosity:

$$q_{s,i}^k = q_{w,i}^k \cdot (1-\varepsilon_i^k), \quad (13)$$

$$q_{g,i}^k = q_{w,i}^k \cdot \varepsilon_i^k, \quad (14)$$

where ε_i is the porosity in the i-th cell, determined taking into account the values of the particle contents along the chain describing the bed (Eq. (1)-(2)).

The mechanism of heat transfer between the surface and the fluidized bed is quite controversial [27-28], which does not allow implementing some generally accepted rules for its calculation. Equations (13)-(14) expresses model representations of heat transfer at the local level and are used in the work to account for the heterogeneity of the phase distribution along the heat exchange surface.

To estimate the value of the gas-particle heat transfer coefficient α_i, correlation dependences were also used for the defined heat transfer criteria [24, 28]:

$$\left.\begin{aligned} \mathrm{Nu}_i^k &= 0{,}016\cdot\left(\frac{\mathrm{Re}_i^k}{\varepsilon_i^k}\right)^{1/3}\cdot \mathrm{Pr}^{0{,}33}, \text{for } \frac{\mathrm{Re}_i^k}{\varepsilon_i^k}<200 \\ \mathrm{Nu}_i^k &= 0{,}4\cdot\left(\frac{\mathrm{Re}_i^k}{\varepsilon_i^k}\right)^{2/3}\cdot \mathrm{Pr}^{0{,}33}, \text{for } \frac{\mathrm{Re}_i^k}{\varepsilon_i^k}>200 \end{aligned}\right\}, \quad (15)$$

For the calculation of the value of the heat transfer coefficient between the gas suspension flow and the wall α_w, we proceed from models describing the gas flow with a wall, which is modified as follows [28]:

$$\mathrm{Nu}_{w,i}^k = \mathrm{Nu}_{g,i}^k\cdot\left[1+6{,}7\cdot \mathrm{Re}_0^{-0{,}3}\times\left(\frac{\mu\cdot c_p}{c_g}\right)\cdot\left(\mathrm{Re}_i^k\right)^{1/3}\right], \quad (16)$$

where Nu_g is the Nusselt number for the heat exchange process between the gas flow and the wall, Re_0 is the number for the gas movement process in the empty section of the apparatus, μ is the dynamic viscosity of the carrier medium, c_p and c_g are the heat capacities of the particle and gas material.

To calculate the heat transfer coefficient from the wall to the gas coolant flowing in it, a generalized empirical ratio is used [29]:

$$\mathrm{Nu}_{g,i}^k = 0{,}021\cdot \mathrm{Re}_0^{0{,}8}\cdot \mathrm{Pr}_{0,i}^{0{,}43}\cdot\left(\frac{\mathrm{Pr}_{0,i}}{\mathrm{Pr}_w}\right)^{0{,}25}, \quad (17)$$

where Pr_0 and Pr_w are the values of the Prandtl numbers at gas temperatures (depends on the cell) and the wall (considered constant).

The equations (11)-(14) determine the amounts of heat transferred per calculated step between the phases of the bed and the heat transfer between the bed phases and the wall surface, and the ratios (15)-(17) allow us to estimate the rates of these transfers. In general, the thermal state of the phases is characterized by the thermal state vectors $\mathbf{Q_p}$ and $\mathbf{Q_g}$ (similar in structure to the corresponding vectors $\mathbf{S_p}$ and $\mathbf{S_g}$). The evolution of the thermal state vectors, taking into account all heat transfer processes, can be written as:

$$\mathbf{Q}_p^{k+1} = \mathbf{P}_p^k\,\mathbf{Q}_p^k + q^k + q_s^k; \quad (18)$$

$$\mathbf{Q}_g^{k+1} = \mathbf{P}_g^k\,\mathbf{Q}_g^k + \mathbf{Q}_{gf} - q^k + q_g^k, \quad (19)$$

where $\mathbf{Q_g}$ is the vector of heat for gas; $\mathbf{Q_p}$ is the vector of heat for particles, $\mathbf{Q_{gf}}$ is the vector of heat source for gas has one non-zero element, the value of which is equal to the heat coming in with the air ($\mathbf{Q_{gf}=T_0\cdot c_g\cdot S_{gf}}$, where $\mathbf{T_0}$ is the temperature of the gas supplied to the apparatus, c_g is the heat capacity for the gas phase). The evolution of vectors $\mathbf{Q_g}$ and $\mathbf{Q_p}$ also entails an adjustment of temperature fields and a change in the rate of heat and mass transfer. The adjusted values of the elements of these vectors can be found as follows:

$$\mathbf{T}_p^{k+1} = \mathbf{Q}_p^{k+1}\mathbf{./\,[c.*\,S_p]}, \quad (20)$$

$$\mathbf{T}_g^{k+1} = \mathbf{Q}_g^{k+1}\mathbf{./\,[c_g.*\,S_g]}, \quad (21)$$

$$\mathbf{T}_s^{k+1} = \mathbf{[Q}_g^{k+1} + \mathbf{Q}_p^{k+1}\mathbf{]./\,[}c_{eff}\mathbf{*\,(S}_g + \mathbf{S}_g\mathbf{)]}, \quad (22)$$

where c_{eff}, c_p are the heat capacities for the gas-solid suspension and solids respectively.

The value of the effective thermal capacity of the gas-solid suspension c_{eff} was estimated based on the assumption that the contribution to the total value of the thermal capacity is proportional to the proportion of the component (additive rule).

$$c_{eff} = [c_p\cdot S_p + c_g\cdot S_g]/V_{cell}, \quad (23)$$

where V_{cell} is the cell volume.

It is necessary to remain within the limits of the stability of the scheme and the solution algorithm when performing recurrent calculation procedures, for which well-known criteria have been developed [31-32]. For Markov processes with generation and death these issues are also acute [32-33]. As practically important considerations, it is necessary to take into account the fact that the equations for calculating the transfer probabilities (9)-(10) contain the relations set during the numerical experiment «$\Delta t/\Delta x^2$» и «$\Delta t/\Delta x$». These relations should remain in such ranges that the natural requirements for probabilities are achieved. Based on this requirements it is possible to formulate the formal criterion for computational stability of recurrent procedures as follows:

$$0 \le \frac{|w_i - V_{si}|}{\Delta x}\Delta t + \frac{2\cdot D\cdot \Delta t}{\Delta x^2} < 1. \quad (24)$$

Procedures and methods of the experimental study

The laboratory setup has been developed and constructed for the experimental study of interfacial heat transfer in the process of fluidization of dispersed material. The scheme and general view of the installation are shown in Figures 1b and 1c respectively. Similar installations with heat supply to the fluidized bed from the heating jacket have been used recently in engineering practice [30]. The main element of the laboratory unit is a cylindrical quartz glass apparatus 1. The diameter of the glass column is 50 mm, the height of the working area is 300 mm. The glass column is closed from the outside with thermal insulation 2. The dispersed material is loaded onto the gas distributor

grid 3, located in the lower part of the reactor. The stand is equipped with an electric heater 4 to provide the necessary temperature regime. Quartz sand with an average fraction size of 1 mm was used as a dispersed material. The bulk density of quartz sand is 1450 kg/m^3 with a true density of about 2600 kg/m^3. A fixed bed of quartz sand in a loose-filled state in the apparatus occupied a height of 60 mm. The height of the fluidized bed ranged from 80-90 mm. The air was pumped into the apparatus by a compressor to provide a fluidization state of the bulk medium. The superficial air velocity was 1,5 m/s. The air temperature at the entrance to the apparatus was maintained at 20 °C. The temperature of the inner wall of the device was 925 °C. After loading the material and starting the fluidization process, temperature sensors were placed in the apparatus and temperature records were taken at various heights from the level of the gas distribution grid using a multi-channel temperature meter «Sosna-004». The sensor of the thermocouple was protected by a cap, which minimized the likelihood of contact with the grains of the bulk medium, the temperature of which was assumed to be the same in terms of the volume of the bed at any given time. The temperature of the particles was determined only at the end of the experiment when unloading the attachment from the apparatus. The experiment was repeated three times, while the stopping time of the process was 2 min, 5 min and 7 min respectively. The gas temperatures in all three experiments differed by less than 5%. The temperature of the bulk medium (for the sample as a whole) in the experiments was 486 °C, 496 °C and 493 °C.

RESULTS AND DISCUSSION

The device was quickly brought to a certain stationary temperature regime before the experiments are carried out. For devices with a fluidized bed in which the material is heated by a gas agent, the steady-state mode of operation is obvious, the temperature of the particles becomes equal to the temperature of the gas phase. For a device with a heating jacket, the stationary mode is not quite obvious, since it depends on the residence time of the phases inside the device, on the characteristics of the bed and the properties of solids.

The results of the performed computational and experimental studies are presented in Fig. 2-3. With a known distribution of the volume content of particles along the length of the chain of cells (described by the vector $\mathbf{S}_p$) it is not difficult to proceed to the description of the distributions of the mass concentration of particles or to the local porosity of the bulk medium along the height of the apparatus.

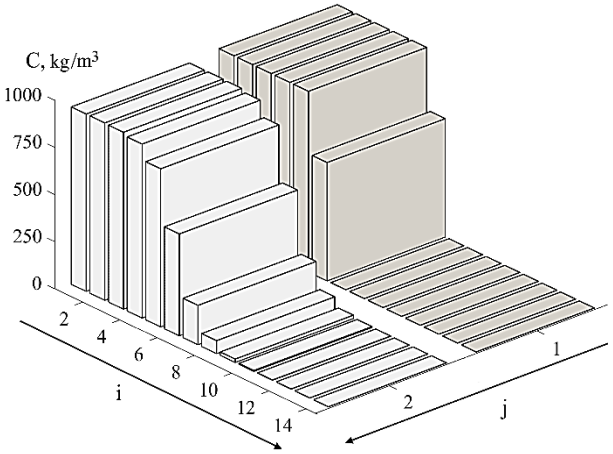


Fig. 2. Distribution of the mass content of the solid phase along the height of the apparatus: j is the state of the bed (1 – fixed; 2 – fluidized); i is the cell number (height of each cell Δx = 10 mm)
Рис. 2. Распределение массового содержания твердой фазы по высоте аппарата: j – состояние слоя (1 – рыхлонасыпанное; 2 – псевдоожиженное); i – номер ячейки (высота каждой Δx = 10 мм)

Fig. 2 shows the calculated distributions of the solid phase mass concentration over the column height for the fixed bed (j = 1) and its fluidized state (j = 2). As it can be seen, the calculated expansion of the bed corresponds to the values of the height of the bed that were observed during the experiment. The distribution of the solid phase concentration is qualitatively consistent, and the prognostic capabilities of the described approach to calculation were tested in the framework of previous works [19, 22]. It should be noted that the diffusion spread of particles at d > 0 provides the presence of material in all cells of the chain (in any volume of the apparatus), therefore, the axial coordinate value below which 95% of the material is concentrated is conventionally taken as the height of the bed (cell number i = 8 in Fig. 2). At the same time, the cells above this conditional boundary of the bed also contain an insignificant number of particles, therefore, for example, the temperature of the material is calculated for the cells of the bed and with numbers i > 8 (Fig. 3).

Fig. 3a shows the results of comparing the calculated values of the phase temperatures of the bed with the experimental values (markers). As it can be seen, the calculated and experimental data regarding the local values of the gas phase temperatures are in good agreement. The calculation (dotted line) shows a slight increase in the temperature of the particles along the height of the bed (from 476 °C in the lower cell to 493 °C in the upper cell), which generally corresponds to the temperature values of the material in the attachments unloaded from the apparatus at the end of the experiment (486 °C, 496 °C and 493 °C).

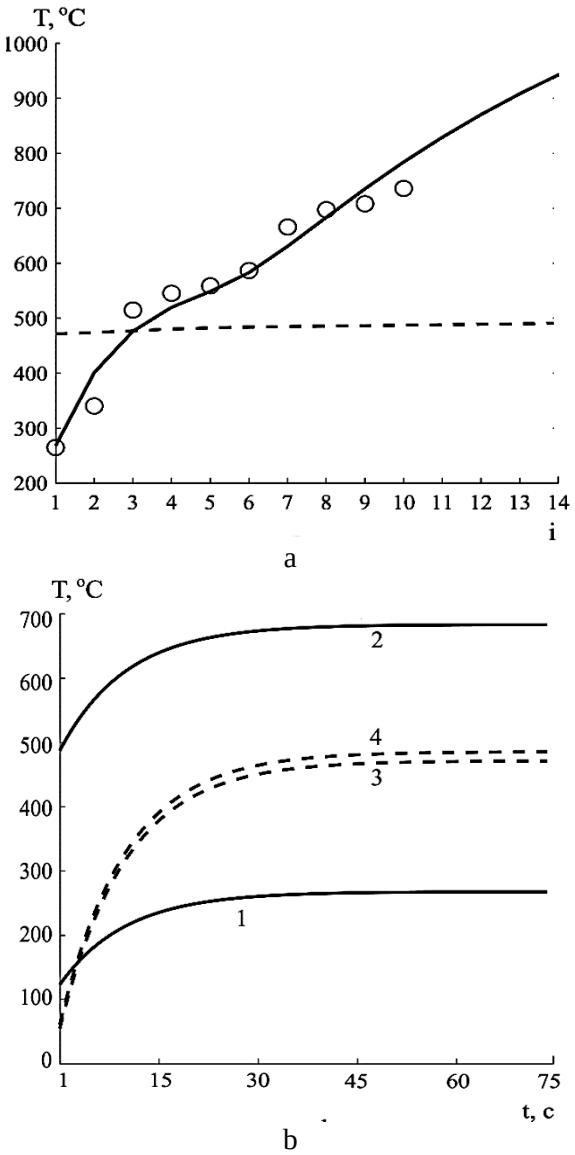


Fig. 3. a – temperature distributions of the bed phases (lines – calculation (dotted line is the gas temperature, solid line is the gas-solid temperature), markers – experiment) according to the height of the apparatus (i is the cell number); б – calculated kinetics of heating of particles and gas to steady values: solid lines – gas phase, dotted lines – solid phase (1 and 3 – in the lower section of the bed (i=1); 2 and 4 – in the upper section of the bed (i=8))

Рис. 3. а – распределения температур фаз слоя (линии – расчет (пунктир – температура газа, сплошная линия – температура газовзвеси), маркеры – эксперимент) по высоте аппарата (i – номер ячейки); б – расчетная кинетика прогрева частиц и газа до установившихся значений: сплошные линии – газовая фаза, пунктирные линии – твердая фаза (1 и 3 – в нижнем сечении слоя (i=1); 2 и 4 – в верхнем сечении слоя (i=8))

Fig. 3b shows the kinetics of heating particles and gas to steady values. As follows from the figure, the temperatures of the gas phase in different sections of the height of the apparatus differ significantly (solid lines 1 and 2 in Fig. 3b), at the same time, the temperatures of the solid phase in the lower and upper sections of the bed differ slightly (dotted lines 3 and 4 in Fig. 3b) due to particle mixing. Also from Fig. 3b it follows that the steady state occurs in a short period of time about 40-45 s.

CONCLUSIONS

In the course of this computational and experimental study, a mathematical model of the functioning of the fluidized bed apparatus with a heating jacket was formed, its parametric identification and verification was performed by comparing the calculated forecasts with the results of a full-scale experiment. The obtained computational and experimental results clearly show the need to consider the technological process in such apparatuses as objects with distributed characteristics. Thus, the proposed mathematical model has sufficient predictive efficiency for engineering tasks and can be considered as a basis for constructing a computer method for calculating heat exchangers with a heating jacket using the fluidization technique of bulk media.

The authors declare the absence a conflict of interest warranting disclosure in this article.

Авторы заявляют об отсутствии конфликта интересов, требующего раскрытия в данной статье.

NOMENCLATURE

A – cross section of discharge opening, m^2
C – particle concentration, kg/m^3
c_{eff} – heat capacity for the gas-solid suspension, J/[K·kg]
c_p – heat capacity for solid, J/[K·kg]
c_g – heat capacity for gas, J/[K·kg]
D – macrodiffusion coefficient, m^2/s
d – diffusion transition probability
F – particle surface, m^2
F_w – wall surface, m^2
G_f, G_d – feed and discharge flow rate, kg/s
h – total number of cells in bed
i – cell number
k – transition number
m – discharge cell number
n – total number of cells in apparatus
Nu_g – Nusselt number for the gas-wall heat exchange process
Nu – Nusselt number for the gas-solid heat exchange process
P_p – matrix of transition probabilities for solid phase
P_p – matrix of transition probabilities for gas phase
Pr_0 – Prandtl number at gas temperature
Pr_w – Prandtl number at wall temperature
p_s – probability of remaining in the cell
p_d – probability of going down to the next cell
p_u – probability of going up to the next cell

Q_p – vector of heat distribution along the chain for solid phase, J
Q_g – vector of heat distribution along the chain for gas phase, J
Q_{gf} – the vector of heat source for gas, J
q – amount of heat transferred between particles and gas during transition duration, J
q_s – amount of heat transferred from the wall to the particles during transition duration, J
q_g – amount of heat transferred from the wall to the gas phase during transition duration, J
q_w– amount of heat transferred from the wall to the gas-solid suspension during transition duration, J
Re_0 – Reynolds number for the superficial gas velocity
Re – Reynolds number for the local gas velocity in a cell
S_p – vector of particle content in cells, kg
S_g– vector of gas content in cells, kg
S_{gf} – source vector of the gas flow, kg
S_{max} –maximum particle content in a cell, kg
T_w – temperature of the wall, °C
T_p – solid temperature, °C
T_g – gas temperature, °C
T_w – gas-solid suspension temperature, °C
Δt – transition duration, s
t_k – the physical time corresponding to the k-th duration, s
v – convection transition probability
V_S – particle settling velocity, m/s
V – particle velocity in a cell, m/s
V_{cell} – cell volume, m^3
W_0 – superficial gas velocity (in empty area), m/s
w – local gas velocity in a cell, m/s
Δx – cell height, m

Greek letters

α – gas-solid heat transfer coefficient, $J \cdot s^{-1} \cdot m^{-2} \cdot K^{-1}$
α_w – suspension-wall heat transfer coefficient, $J \cdot s^{-1} \cdot m^{-2} \cdot K^{-1}$
μ – the dynamic viscosity of the carrier medium, $N \cdot s \cdot m^{-2}$
ε – porosity

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