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**МАРКОВСКАЯ МОДЕЛЬ ПЕРИОДИЧЕСКОЙ СУШКИ ЧАСТИЦ В КОНИЧЕСКОМ ПСЕВДООЖИЖЕННОМ СЛОЕ**

*Целью исследования является разработка модели для описания гидродинамики, тепло- и массопереноса в коническом псевдооживленном слое с частицами, у которых значительно меняются свойства. Предлагаемая модель построена на основе теории цепей Маркова, а монофракция кубиков влажного картофеля используется как модельный материал. Сжатие образцов картофеля в процессе сушки было учтено, чтобы повысить адекватность модели. Таким образом, три новых фактора, которые влияют на процесс, приняты во внимание: изменение расходной скорости газа по высоте реактора из-за его конической формы, изменение массы частиц благодаря сушке и изменение размера частиц из-за их сжатия. Слой представлен двумя параллельными цепями ячеек идеального перемешивания: одна цепь для частиц и другая цепь для сушильного газа. Эволюция распределения частиц по своей цепи описана матрицей переходных вероятностей, которая зависит от текущего состояния цепи и меняется с течением времени. Тепло и массоперенос между ячейками обеих соседних цепей описаны обычными уравнениями тепло и массоотдачи. Модель позволяет прогнозировать кинетику сушки, если известны коэффициенты тепло и массоотдачи. Соотношения для расчета коэффициента аэродинамического сопротивления частиц, чисел Нуссельта и Шервуда и коэффициентов сжатия и диффузии заимствованы из литературы. Модель верифицирована на лабораторной сушилке псевдооживленного слоя с конической формой реактора. Получено хорошее совпадение расчетных и экспериментальных данных.*

**Ключевые слова:** псевдооживление, цепь Маркова, вектор состояния, матрица переходных вероятностей, сушка, содержание влаги, кинетика

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## MARKOV CHAIN MODEL OF PARTICULATE SOLIDS BATCH DRYING IN A CONICAL FLUIDIZED BED

*The objective of the present study is to develop a model to describe the hydrodynamics, heat and mass transfer in a conical fluidized bed with particles of strongly variable properties. The proposed model is based on the Markov chains approach, and the wet potato mono-sized cubes are used as the model material. Shrinkage of potato samples during the process of drying is taken into account to improve the adequacy of simulation. Thus, the three new factors that influence the process are taken into account: variation of the superficial gas velocity over the bed height due to its conical shape, variation of particles mass due to drying and variation of particle size due to its shrinkage. The bed is presented as two parallel chains of perfectly mixed cells: one chain for particulate solids, and one chain for the drying gas. The evolution of particulate solids distribution over its chain is describes with the matrix of transition probabilities, which is state dependent and varies with time. The heat and mass transfer between adjacent cells of the both chains is describes with the common relations of heat and mass transfer. The model allows predicting the drying kinetics if the coefficients of heat and mass transfer are known. The correlations to calculate the drag force coefficient, Nusselt and Sherwood numbers, the coefficients of shrinkage and of diffusivity were borrowed from literature. The model was validated at the lab scale fluidized bed dryer with the conical shape of reactor. A good agreement between obtained experimental and calculated results is achieved.*

**Key words:** fluidization, Markov chain, state vector, matrix of transition probabilities, drying, moisture content, kinetics

### 1. INTRODUCTION

Fluidized bed reactors are widely used for thermal and chemical treatment of particulate solids in many industries. In particular, they found broad application in the food industry for drying fruits and vegetables for their preservation. Fruit and vegetables are porous and high moisture containing food products that leads to some specific features in the drying process operation. The point is that properties of such particulate solids change strongly during the process that changes the fluidized bed hydrodynamics, and, in turn, the conditions of particles treatment. If the properties of particles are constant, the transient process of

fluidization is very short but if they continuously vary the transient process lasts during all operating time of particles treatment. It is obvious that this phenomenon is to be taken into consideration if a predictive model of the process is needed.

Fluidization process is commonly organized in a cylindrical apparatus because of its more high predictability. However, fluidization in conical (tapered) vessels has several technological advantages over cylindrical fluidized bed. The tapered form expands range of available operating air velocities. The gas velocity distribution in the cross section of conical bed appeared to be uniform [1]. Intense particles

circulation in such bed occurs due to angled walls. Therefore, fluidization in conical vessels is characterized by more intense mixing of particles compared to cylindrical reactors. The intense mixing depresses solids segregation but the gas flow has less homogeneous structure due to variable cross section area. The decrease of the gas velocity in the direction of gas flow provides adequate fluidization of particles of different size in poly-dispersed systems. Thus, not only particulate solids properties change with time but the superficial gas velocity and other hydrodynamic conditions vary over the bed height that also is to be taken into account [1, 2].

In simulation of tapered fluidized bed the same approaches are used as those that form the basis for modeling cylindrical beds. These computational simulations are usually based on various combinations of Lagrangian and Eulerian approaches [3]. In particular, a discrete model (Lagrangian-Eulerian) to predict particles motion in a pseudo-2D spout fluidized bed and its experimental verification were presented in the paper [4]. The Eulerian-Eulerian approach was successfully used in [5] to describe dynamics of spouted beds with conical-cylindrical and conical geometries. Actually, it can be noted that there exist a lot of works devoted to fluidization technologies in cylindrical beds but small minority of research projects were devoted to the tapered ones. Additionally, the relevance of fluidization technology to food processing is still at an early stage [6]. The design of such fluidized bed reactors are based mostly on empirical basis, and modeling approaches based on different length and time scales still require development [3, 6].

According to our viewpoint the approach based on the theory of Markov chains is one of the most effective tools to model the process. This approach was first used in [7] to describe the process of fluidization and then it was successfully used for simulating of a wide range of processes in particle technology [8, 9]. The present study has the main objective to describe particulate solids drying in a tapered fluidized bed on the basis of the Markov chains approach.

## 2. THEORY

The present study is the logical continuation of the approach developed in our previous work [10]. The computational scheme of the process is presented schematically in Fig. 1.

It is a 1D model, in which the operating volume of reactor is divided into  $n$  perfectly mixed cells in axial direction. The height of each cell is  $\Delta x = H/n$  where  $H$  is the height of the reactor. The process is

observed at the discrete moments of time  $t_k = (k-1)\Delta t$  where  $\Delta t$  is the time step and  $k$  is the number of transition. Two parallel chains of cells are separated in the operating volume. One of them belongs to particulate solids, and another one belongs to the gas flow. Particles and gas can travel along their chains and exchange heat and mass according to corresponding driving forces.

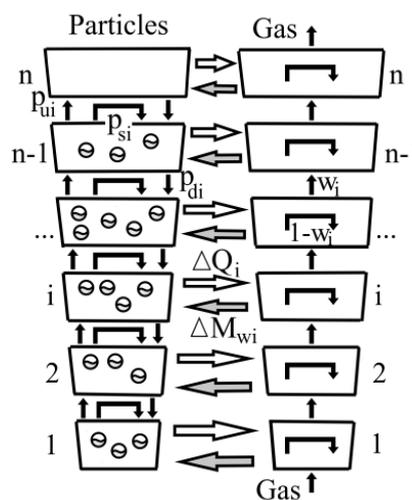


Fig. 1. Computational scheme of the process and its cell presentation

Рис. 1. Расчетная схема процесса и ее представление в виде ячеек

At no gas flow action the particles occupy several bottom cells. Then the air flow begins to act with the superficial velocity  $W_0$  related to the empty cross section of the reactor. The concentration of particles is high at this time, and the local velocity  $W_i$  of flow around particles is much higher than  $W_0$  because the voids between particles are rather small. If this velocity is higher than the particle settling velocity, particles begin ascending with the velocity  $V_i = W_i - V_s$  and occupy upper cells. Their concentration is getting smaller, that leads to the decrease of  $W_i$  and  $V_i$  respectively. Finally, at a certain level (in the  $h$ -th cell)  $V_i$  becomes equal to zero that corresponds to the upper level of the bed (if the equilibrium between  $W_i$  and  $V_s$  is not reached in the cell  $n$ , the bed will be blown out, and the stable bed cannot exist at such regime). The transitions caused by interaction between gas flow and particles can be called the convection transitions. If the equilibrium is reached in the cell  $h$ , the bed becomes "locked" at this level.

The particles volume content distribution over the cells of the chain can be presented as the column state vector  $S_p = \{S_{pi}\}$  where  $i = 1, 2, n$  from the bed bottom. Its evolution can be described by the nonlinear recurrent matrix equality:

$$\mathbf{S}_p^{k+1} = \mathbf{P}_p^k \mathbf{S}_p^k, \quad (1)$$

where  $\mathbf{P}_p$  is the matrix of transition probabilities that depends itself on the current state vector,  $k$  is the

$$\mathbf{P}_p(\mathbf{S}_p^k) = \begin{bmatrix} 1 - v_1(\mathbf{S}_{p1}^k) - d & d & 0 & 0 & \dots \\ v_1(\mathbf{S}_{p1}^k) + d & 1 - v_2(\mathbf{S}_{p2}^k) - 2d & d & 0 & \dots \\ 0 & v_2(\mathbf{S}_{p2}^k) + d & 1 - v_3(\mathbf{S}_{p3}^k) - 2d & d & \dots \\ 0 & 0 & v_3(\mathbf{S}_{p3}^k) + d & 1 - v_4(\mathbf{S}_{p4}^k) - 2d & \dots \\ \dots & \dots & \dots & \dots & \dots \end{bmatrix}, \quad (2)$$

where  $v_i = V_i \Delta t / \Delta x$  is the convection transition probability caused by the gas-particle interaction, and  $d = D \Delta t / \Delta x^2$  is the diffusion transition probabilities caused by the particle-particle interaction and gas flow turbulence ( $D$  is the dispersion coefficient). The diffusion transition probabilities are supposed to be symmetrical and have no influence of the bed expansion that is completely defined by the convection transitions. Thus, the key problem in making the model suitable for engineering application is how to determine  $V_i$  and  $D$ .

The value of  $W_i$  can be defined as the function of solids concentration obtained on the basis of the computational scheme presented in our previous work [10]:

$$W_i = \frac{W_{0,i}}{1 - p \left( \frac{3S_{p,i}}{4p} \right)^2} = \frac{W_{0,i}}{1 - p \left( \frac{S_{p,i}}{8S_{\max,i}} \right)^2}, \quad (3)$$

where  $S_{\max,i}$  is the maximum possible content of particles in the  $i$ -th cell that can be easily defined experimentally for the random packing of particles.

After the gas supply begins the value of  $W_i$  is decreasing and at last becomes equal to the particle settling velocity  $V_s$ . The settling velocity  $V_s$  can be found from the following equation of particle equilibria in the upstream flow:

$$m_p g = C_d \cdot f_p \cdot \rho_g \cdot V_s^2 / 2, \quad (4)$$

where  $m_p$  is the particle mass,  $C_d$  is the drag force coefficient,  $f_p$  is the particle cross section area,  $\rho_g$  is the gas density. In order to calculate the velocity  $V_s$  it is necessary to have a reliable fitting formula for the drag force coefficient  $C_d$ .

The gas motion through its chain is described by with the recurrent matrix equality:

$$\mathbf{S}_g^{k+1} = \mathbf{P}_g^k \mathbf{S}_g^k + \mathbf{S}_{gf}, \quad (5)$$

where  $\mathbf{P}_g$  is the matrix of transition probabilities for gas (it contains the part of gas that transits from the cell  $i$  to the cell  $(i+1)$  during  $\Delta t$ );  $\mathbf{S}_g$  is the column vector of gas volume content in the cells;  $\mathbf{S}_{gf}$  is the column vector of gas source (it contains the only non-zero element if the gas comes only through the air distributor).

number of time transition of duration  $\Delta t$ . The matrix  $\mathbf{P}_p$  has the following form:

The gas and particles motion along the chains are described by equations (1) and (7). The exchange of the mass and heat content between corresponding cells of the chains during the  $k$ -th time step can be described by the next balance equations:

$$\mathbf{M}_{wp}^{k+1} = \mathbf{P}_p^k (\mathbf{M}_{wp}^k - \mathbf{k}_w \cdot \mathbf{F}^k \cdot (\mathbf{p}_{ws}^k - \mathbf{p}_{wg}^k) \Delta t) \quad (6)$$

$$\mathbf{M}_{wg}^{k+1} = \mathbf{P}_g^k (\mathbf{M}_{wg}^k + \mathbf{k}_w \cdot \mathbf{F}^k \cdot (\mathbf{p}_{ws}^k - \mathbf{p}_{wg}^k) \Delta t + \mathbf{M}_{wgf}) \quad (7)$$

$$\mathbf{Q}_p^{k+1} = \mathbf{P}_p^k (\mathbf{Q}_p^k + \alpha \cdot \mathbf{F}^k \cdot (\mathbf{T}_g^k - \mathbf{T}_p^k) \Delta t - \mathbf{r} \mathbf{k}_w \cdot \mathbf{F}^k \cdot (\mathbf{M}_{wp}^k - \mathbf{M}_{wg}^k) \Delta t) \quad (8)$$

$$\mathbf{Q}_g^{k+1} = \mathbf{P}_g^k (\mathbf{Q}_g^k - \alpha \cdot \mathbf{F}^k \cdot (\mathbf{T}_g^k - \mathbf{T}_p^k) \Delta t + \mathbf{Q}_{gf}), \quad (9)$$

where  $\mathbf{M}_{wp}$  is the state vector of moisture content in solid;  $\mathbf{k}_w$  is the drying rate coefficient;  $\mathbf{F}$  is the vector of solid surface;  $\mathbf{p}_{ws}$  is the vector of partial pressure of water vapour on the surface of particle;  $\mathbf{p}_{wg}$  is the vector of partial pressure of water vapour in gas;  $\mathbf{M}_{wg}$  is the vector of moisture content in gas,  $\mathbf{M}_{wgf}$  is the vector of moisture source for gas;  $\mathbf{Q}_g$  is the vector of heat content for gas;  $\mathbf{Q}_p$  is the vector of heat content for particles ( $\mathbf{Q}_p = \mathbf{T}_p \cdot \mathbf{c} \cdot \rho$  where  $\mathbf{c}$  is the vector of heat capacities of moist particles,  $\rho$  is the vector of densities of moist particles;  $\alpha$  is the vector of the local heat transfer coefficients;  $\mathbf{T}_g$  is the vector of gas temperature;  $\mathbf{T}_p$  is the vector of solid temperature;  $r$  is the latent heat of vaporization;  $\mathbf{Q}_{gf}$  is the vector of heat source for gas.

The process of gas-solid mass exchange provides the changing of particles densities. The current densities of particles is defined by the equation:

$$\rho^k = (\mathbf{M}_{wp}^k + \rho_p \mathbf{S}_p^k) / \mathbf{S}_p^k, \quad (10)$$

where  $\rho_p$  is the density of dry particles.

The moisture content distributions over the chains are calculated as follows:

$$\mathbf{X}_p^k = \mathbf{M}_{wp}^k / (\rho_p \mathbf{S}_p^k) \quad (11)$$

$$\mathbf{X}_g^k = \mathbf{M}_{wg}^k / (\rho_g \mathbf{S}_g^k), \quad (12)$$

where the dry gas density  $\rho_g$  is calculated using the gas state equation.

It must be noted that the dimensions of food particles are changing strongly during drying that influences on their settling velocity. The shrinkage of particles can be described on empirical basis.

*Identification of the model parameters*

The formula for the drag force coefficient  $C_d$  was taken from our previous work [10]:

$$C_d(Re_p, Ar) = \frac{24}{Re_p} + \frac{Ar}{Re_p^{1.96}} \quad (13)$$

The relationship for calculation of the effective moisture diffusivity of potato was borrowed from the work [11]:

$$D = D_0 \exp\left(-\frac{X_0}{X}\right) \exp\left(-\frac{T_0}{T}\right), \quad (14)$$

where  $D_0$ ,  $X_0$  and  $T_0$  are the empirical parameters.

Dependence of saturated vapor pressure on temperature (i.e., the elements of the vector  $\mathbf{p}_{ws}$ ) was used in the following form [11]:

$$\ln(p_{wsi}^k) = A - \frac{B}{C + T_{pi}^k}, \quad (15)$$

where  $A = 16.377$ ,  $B = 3878.82$ ,  $C = 229.86$  for water vapor.

The partial pressure of water vapor in the humid air (i.e., the elements of the vector  $\mathbf{p}_{wg}$ ) was calculated with the formula presented in [12]:

$$p_{wgi}^k = 4.61 c_{wgi}^k (273 + T_{gi}^k) 10^3, \quad (16)$$

where  $\rho_{wgi}^k$  is the absolute humidity of air in the  $i$ -th cell ( $\text{g}/\text{cm}^3$ ).

The following criterial correlations proposed by Bird, R.B. et al. [13] were used to calculate the transfer coefficients:

$$Nu = 2.0 + 0.6 Re_p^{1/2} Pr^{1/3}, \quad (17)$$

$$Sh = 2.0 + 0.6 Re_p^{1/2} Sc^{1/3}, \quad (18)$$

where  $Re_p$  is Reynolds number,  $Sc$  is Schmitt number,  $Pr$  is Prandtl number.

The shrinkage of potato particles was calculated as the function of its moisture contents [14]:

$$V_m^k = V_d(1 + \beta_v X_p^k), \quad (19)$$

where  $V_m$  is the current volume of particle,  $V_d$  is the volume of dry particle,  $\beta_v$  is the shrinkage coefficient (for potato  $\beta_v = 0.625$ ) [14].

3. MATERIALS AND METHODS

The drying experiments were carried out at a lab scale batch conical fluidized bed dryer. The experimental set-up is shown schematically in Fig. 2. The main parts of the set-up are: the fan 1, the gas heater 2, the rotameter 3, the temperature and gas humidity measuring system 4, the conical transparent reactor 5, and the removable basket with the perforated gas distributor 6. The removable basket connected with perforated stainless gas distributor, is inserted into reactor. This allows periodical removing the bed hold-up to measure average moisture content in it. Heated air

was the drying agent. The potato cubes with the side  $H_p = 5 \text{ mm}$  were used as the test material. About  $40 \pm 1 \text{ g}$  of potato cubes were placed into the basket and then into the drying chamber and experiment started. The air was supplied through the rotameter and heater before entering the chamber. The temperature and relative humidity measuring system is based on the digital hygrometer ITV2605 and has four independent sensors. The experiments were performed at the inflow gas temperature  $40 \pm 2 \text{ }^\circ\text{C}$ . The sensors of the hygrometer were placed at 4 positions as it is shown in Fig. 2.

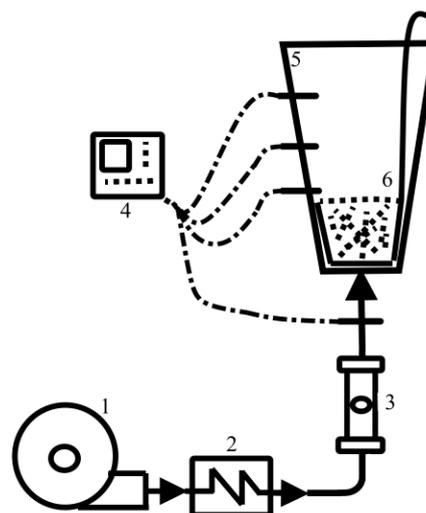


Fig. 2. Schematic presentation of the experimental set up: 1 – fan, 2 – air heater, 3 – rotameter, 4 – digital hygrometer ITV2605, 5 – transparent reactor body, 6 – removable basket

Рис. 2. Схема экспериментальной установки: 1 – вентилятор, 2 – воздушонагреватель, 3 – расходомер, 4 – цифровой гигрометр ИТВ 2605, 5 – прозрачный корпус реактора, 6 – съемная корзина

4. RESULTS AND DISCUSSION

Fig. 3. shows the comparison of calculated and experimental data for evolution of the average moisture content in the solid. It is necessary to note that the drying kinetics is non-linear from the very beginning of the process that means that the first and second drying periods do not exist in their classical form.

Fig. 4 shows the variation of the average fluidized bed expansion with drying time. At the very beginning the bed height grows rapidly due to its primary fluidization. At this period of time the properties of particles remain practically constant. Then the slow growth of the bed height can be observed that occurs due to the particle mass and size reduction during their thermal treatment. The latter leads to the further increase of the bed height approximately two times as much.

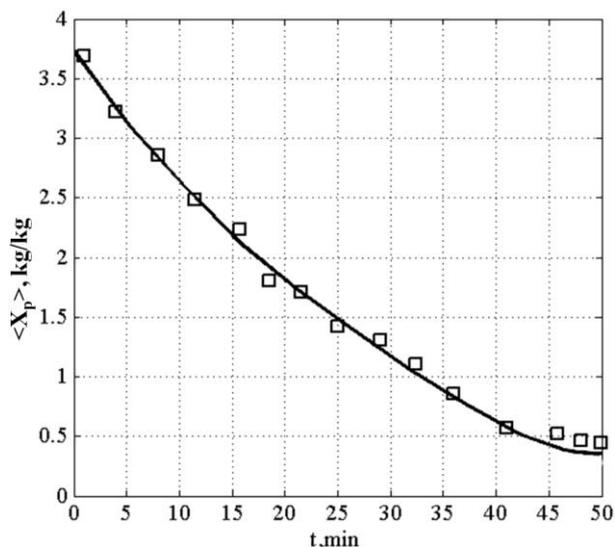


Fig. 3. Kinetics of potato cubes drying (line – calculation, points – experiment)

Рис. 3. Кинетика сушки картофельных кубиков (линия – расчет, точки – эксперимент)

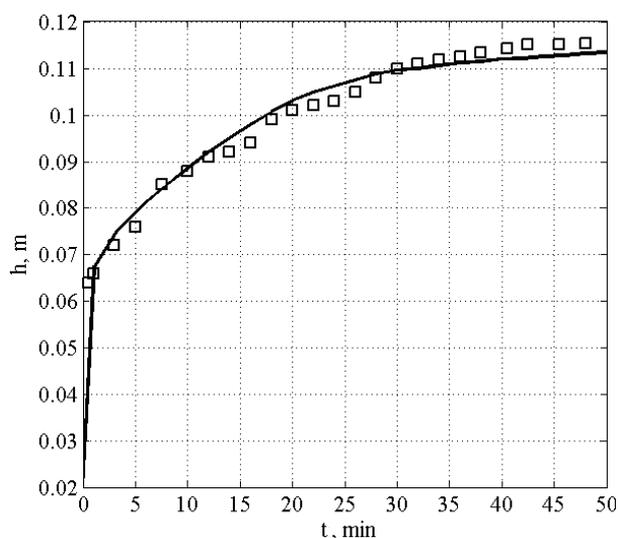


Fig. 4. Bed height versus time (line – calculation, points – experiment)

Fig. 4. Зависимость высоты кипящего слоя от времени (линия – расчет, точки – эксперимент)

The shrinkage of particles was investigated in this experiments too, and it was shown that the linear correlation Eq.(19) describes the process rather well. The average size of potato cube reduced from 5 mm at the beginning to 3.2 mm at the end. It is obvious that such deep decrease of particle size has a strong influence on its settling velocity and, in turn, on the bed height.

Fig. 5 presents the kinetic of relative air humidity variation at three different levels within the fluidized bed. As it can be seen from this figure that the high intense evaporation occurs at the bottom of

the bed, and the air resources for drying are decreasing at the upper regions. The air humidity does not practically change over the bed height after 45 min because the drying is almost ended.

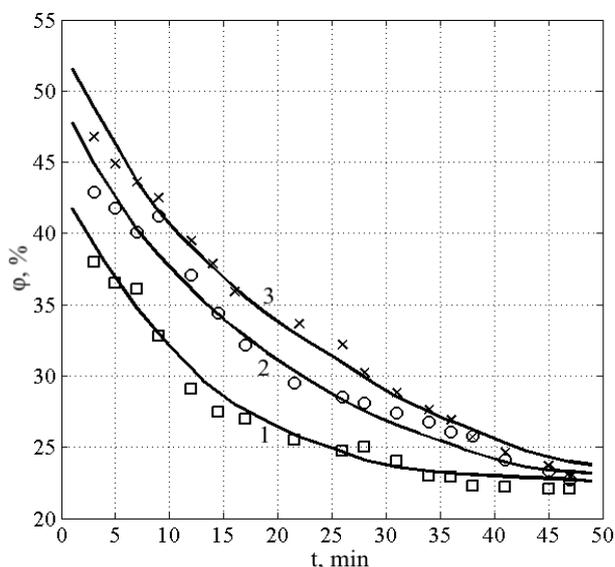


Fig. 5. Relative air humidity versus time at different levels of the reactor (lines – calculation, points – experiment): 1 – 0.015 m, 2 – 0.045 m, 3 – 0.075 m

Рис. 5. Относительная влажность воздуха как функция времени на различных уровнях реактора (линия – расчет, точки – эксперимент). 1 – 0,015 м, 2 – 0,045 м, 3 – 0,075 м

## 5. CONCLUSIONS AND PERSPECTIVES

It is shown, both theoretically and experimentally, that the deep variation of particulate solids properties can occur during its thermal treatment in fluidized bed reactors. In particular, the particle mass can change due to its drying, and the particle size can change due to its shrinkage. This variation can considerably change hydrodynamics of the bed that, in turn, changes the conditions of heat and mass exchange between particles and gas. The bed expansion can grow two times as much, and even more, and the particles can be blown out of the reactor. This can be prevented in a conical fluidized bed reactor that allows keeping the particles with the operating zone at wider range of their settling velocities. The efficient application of a lab scale conical fluidized bed reactor for drying potato cubes as a test material is demonstrated.

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